Sets

Computer Mathematics I

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SETS

A **Set** is a collection of objects.
Examples of sets are:

- the set of integers, denoted by $\mathbb{Z}$.
- the set of nonnegative integers, denoted by $\mathbb{N}$.
- {red, ball, green}

1. The elements of a set need not be related in any way.
2. The objects comprising a set are called its **elements or members**. If x is a member of a set L, then we write $x \in L$. For example $4 \in \mathbb{N}$.
3. A singleton is a set having only one member.
4. A set with no element is called the **empty set**, and is denoted by $\emptyset$.

SETS

There are two ways to display a set:

- Simply list all their elements, separate by commas and include in braces as the following example $\mathbb{N} = \{0, 1, 2, \ldots\}$ or S={a,b,c}
- refer to other sets and the properties that characterize the elements of this set. For example, the set of odd natural numbers can be defined by O ={x : x$\in$ $\mathbb{N}$ and x is not divisible by 2}.

A set with infinitely many elements is called **infinite** and a set with finitely many elements is said to be **finite**. The example of finite set is S={a,b,c} and hence, $\mathbb{N} = \{0, 1, 2, \ldots\}$ is infinite set.

SETS

1. A set X is a **subset** of a set Y, written $X \subseteq Y$, if each element of X is also an element of Y.
2. A set X is a **proper subset** of a set Y, written $X \subset Y$ if X is a subset of Y and X is not equal Y.
3. A set X is **equal** a set Y if X is a subset of Y and Y is a subset of X.
   X = Y if $X \subseteq Y$ and $Y \subseteq X$
4. For any set X, the empty set $\emptyset$ is a subset of X, $\emptyset \subseteq X$ and X is a subset of itself, $X \subseteq X$.

UNION

The union of two sets X,Y is the set of elements which belongs to at least one of them.
$X \cup Y = \{x : \ x \in X \text{ or } x \in Y\}$

For example, the union of a set of even integers and odd integers is the set $\mathbb{Z}$.

Let A={1,2,3} and B={3,4,5}
$A \cup B$={1,2,3,4,5}

INTERSECTION

The intersection of two sets X,Y is the set of elements which belongs to both of them.
$X \cup Y = \{x : \ x \in X \text{ and } x \in Y\}$

For example, the intersection of a set of even integers and odd integers is the empty set $\emptyset$.

Let A={1,2,3} and B={3,4,5}
$A \cap B$={3}

DIFFERENCE

The difference X\Y of two sets X,Y is the set of those elements in X that are not in Y.
$X \setminus Y$ or X-Y = $\{x : x \in X$ and $x \notin Y\}$

For example, the difference of a set of even integers and odd integers is the set of even integers.

Let A={1,2,3} and B={3,4,5}
$A \setminus B$={1,2}

LAW OF SET OPERATIONS

1. Idempotency
$$A \cup A = A$$
$$A \cap A = A$$
2. Commutativity
$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$
3. Associativity
$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$
4. Distributivity
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
5. Absorption
$$A \cap (A \cup B) = A$$
$$A \cup (A \cap B) = A$$
6. DeMorgan's Laws
$$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$$
$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

LAW OF SET OPERATIONS

Suppose there is a big set U such that $A \subseteq$U and $B \subseteq$U.
Let $\overline{A}$ =U$\backslash A$, $\overline{B}$ =U$\backslash B$
Then

- $A \setminus B \;=\; A \cap \overline{B}$
- $\overline{A \cap B} \;=\; \overline{A} \cup \overline{B}$
- $\overline{A \cup B} \;=\; \overline{A} \cap \overline{B}$

POWER SET

The **power set** of a set S, denoted by 2^S, is the set of all subsets of S.
Example:
S = {1,2,3}
2^S={ $\emptyset$, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3} }

If S is a collection of sets then $\bigcup S$ is the set whose elements are the elements of the sets in S.

Example: S = { {1,2}, {2,3} }
$\bigcup S$ = {1,2,3}

PARTITION

A **partition** of a set S is a set Π of subsets of S, i.e. $\Pi \subseteq 2^S$, such that

1. each element of Π is nonempty
2. distinct members of Π are disjoint
3. $\bigcup \Pi = S$

Example S={1,2,3}
Π={ {1}, {2,3} } or
Π={ {1}, {2}, {3} } or
Π={ {1,2,3} }
however, X={ {1,2}, {2,3} } and Y={ $\emptyset$, {1,3}, {2} } are not partitions of S.

EXERCISES

Let A={0,2,4,6}
B={1,3,5}
C={0,1,2,3,4,5,6,7}
D=$\emptyset$
E=$\mathbb{N}$
F={ {0,2,4,6} }

Question:
Find subsets of A =? subsets of B=? subsets of C=? subsets of D=? subsets of E=? and subsets of F=?

EXERCISES

Let U={0,1,2,3,4,5,6,7,8,9}
A={0,1,2,3}
B={0,2,4}
C={0,3,6,9}

Question:
Find $A \cup B$ =?
$A \cap B$ =?
$\overline{A}$=?
$\overline{A \cap B}$=?
$(B \cup C) - A$ =?