Ch14: Binary Search Tree

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Algorithm Analysis and Design
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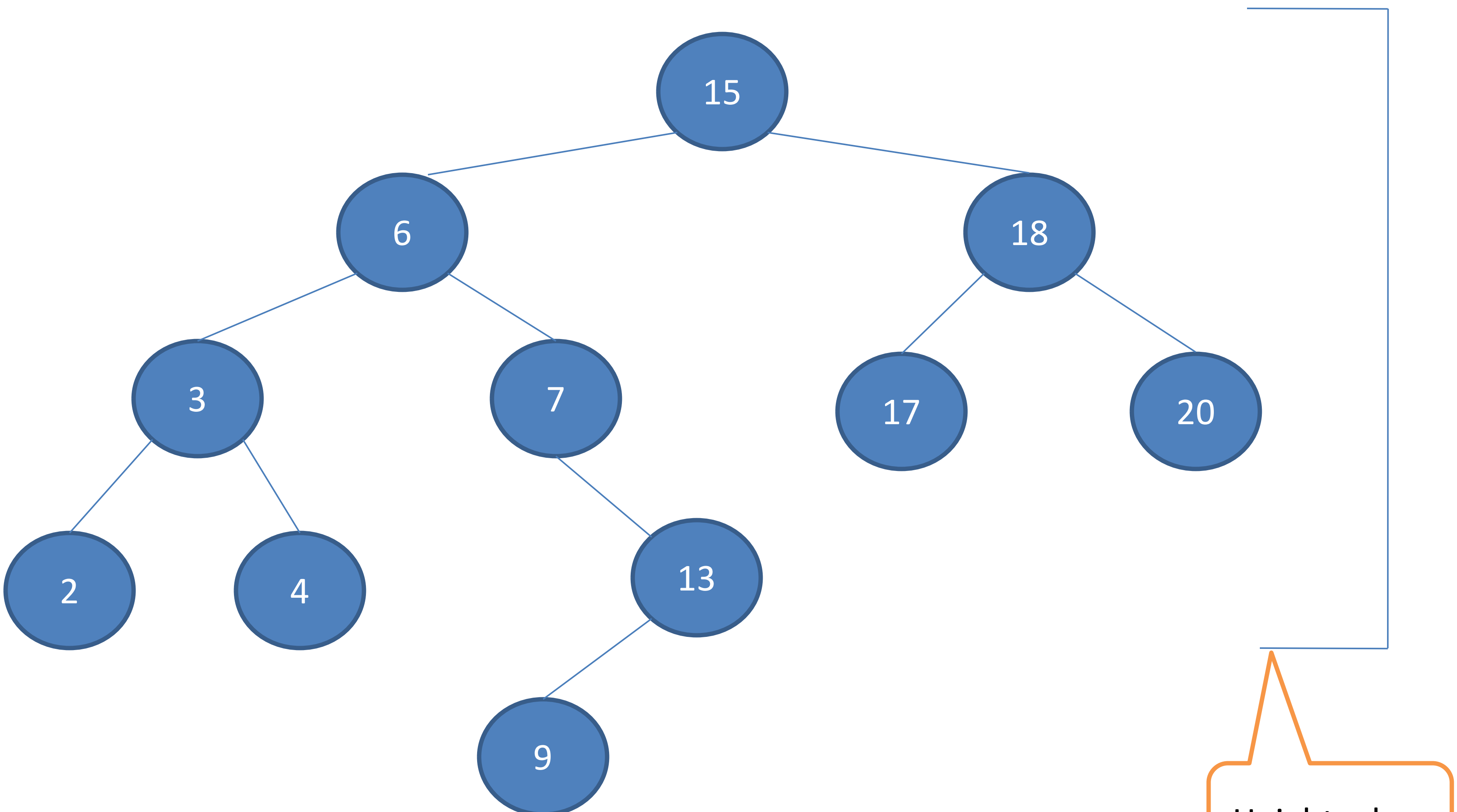
Binary Search Tree

- A binary search tree is organized in a binary tree where each node contains fields left, right, and p that point to the nodes corresponding to its left child, right child and parent, respectively.
- The **binary search tree property**:
  - Let x be a node in a binary search tree. If y is a node in the left subtree of x, then key[y] ≤ key[x]. If y is a node in the right subtree of x, then key[x] ≤ key[y].
- Take time proportional to the height of the tree.
- Expected height of a randomly built binary search tree is O(lg n), so that basic dynamic-set operations on such a tree take $\Theta(\lg n)$ time on average.

Binary Search Tree



Inorder-Tree-Walk(x)

```
if x != NIL
        then Inorder-Tree-Walk(left[x])
        print key[x]
        Inorder-Tree-Walk(right[x])
```

It takes $\Theta(n)$ time

Tree-Search(x,k)

```
if x = NIL or k = key[x]
        then return x
if k < key[x]
        then return Tree-Search(left[x],k)
         else return Tree-Search(right[x],k)
```

Recursion from a path downward from the root of the tree , so the running time is $\Theta(h)$

Iterative-Tree-Search(x,k)

```
while x != NIL and k != key[x]
          do if k < key[x]
                    then x = left[x]
                    else x = right[x]
return x
```

Tree-Minimum(x)

```
while left[x] != NIL
        do x = left[x]
return x
```

Tree-Maximum(x)

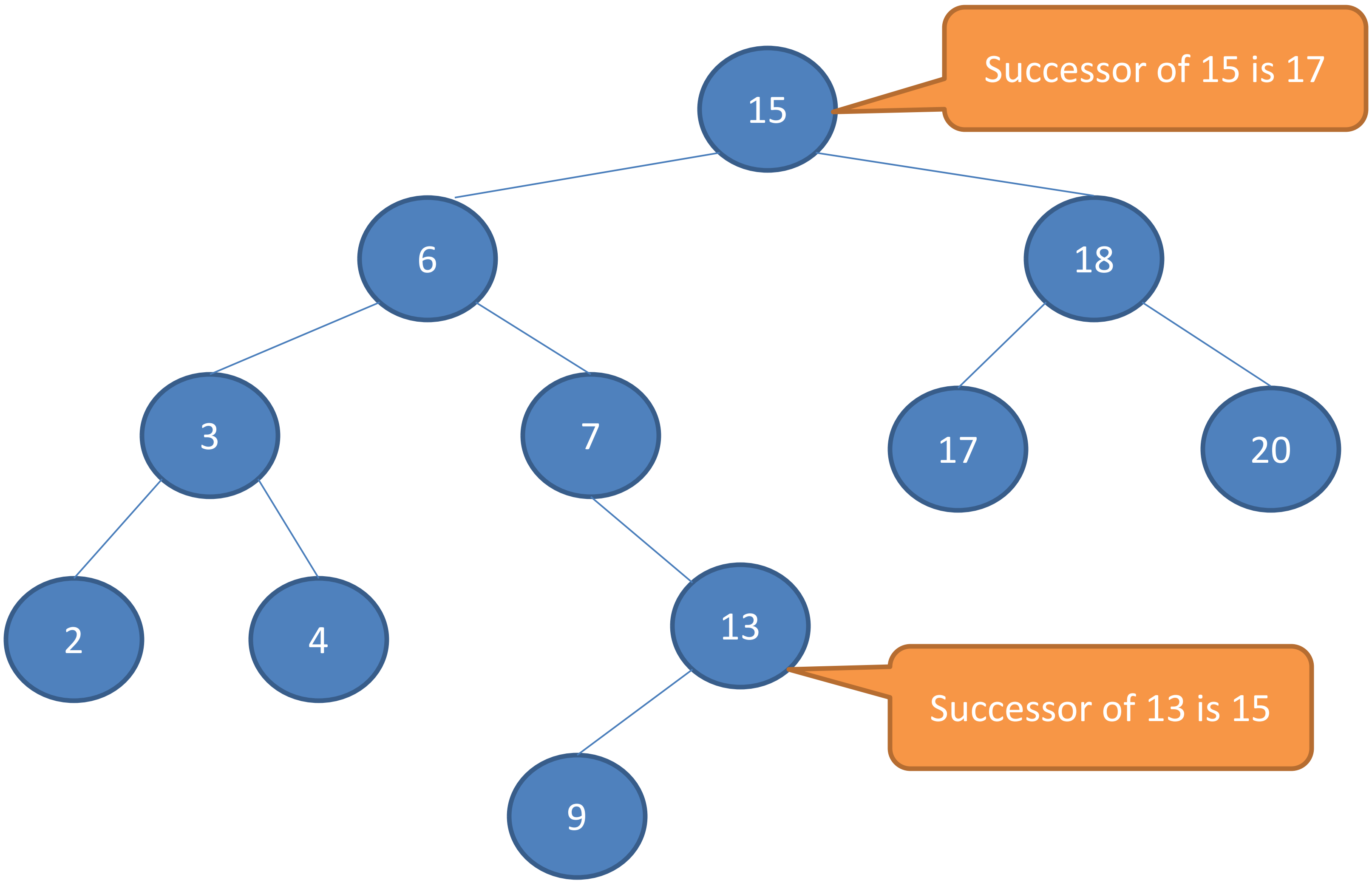
```
while right[x] != NIL
        do x = right[x]
return x
```

Tree-Successor(x)

```
if right[x] != NIL
        then return Tree-Minimum(right[x])
Y=p[x]
while y!=NIL and x = right[y]
        do x=y
        y=p[y]
return y
```

We either follow a path up the tree or follow a path down the tree, so the running time is $\Theta(h)$

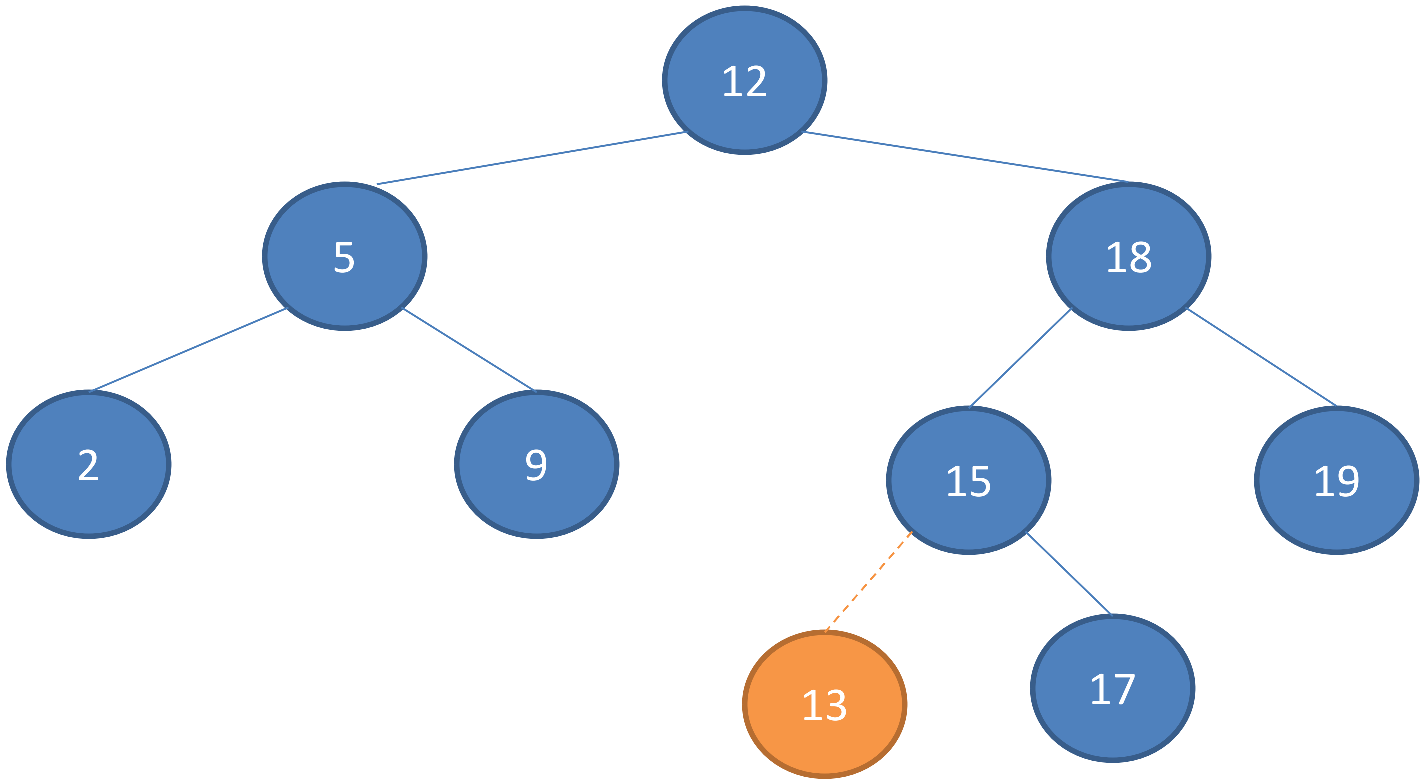
Tree-Successor



Tree-Insert(T,z)

```
y = NIL
x = root[ T ]
while  x != NIL
        do y = x
        if key[ z ] < key[x]
        then x = left[x]
        else x = right[x]
p[z]  = y
if y = NIL
        then root[ T ] = z
        else if key[z] < key[y]
                then left[y] = z
                else right[y] = z
```

Example: Tree-Insert(T,13)

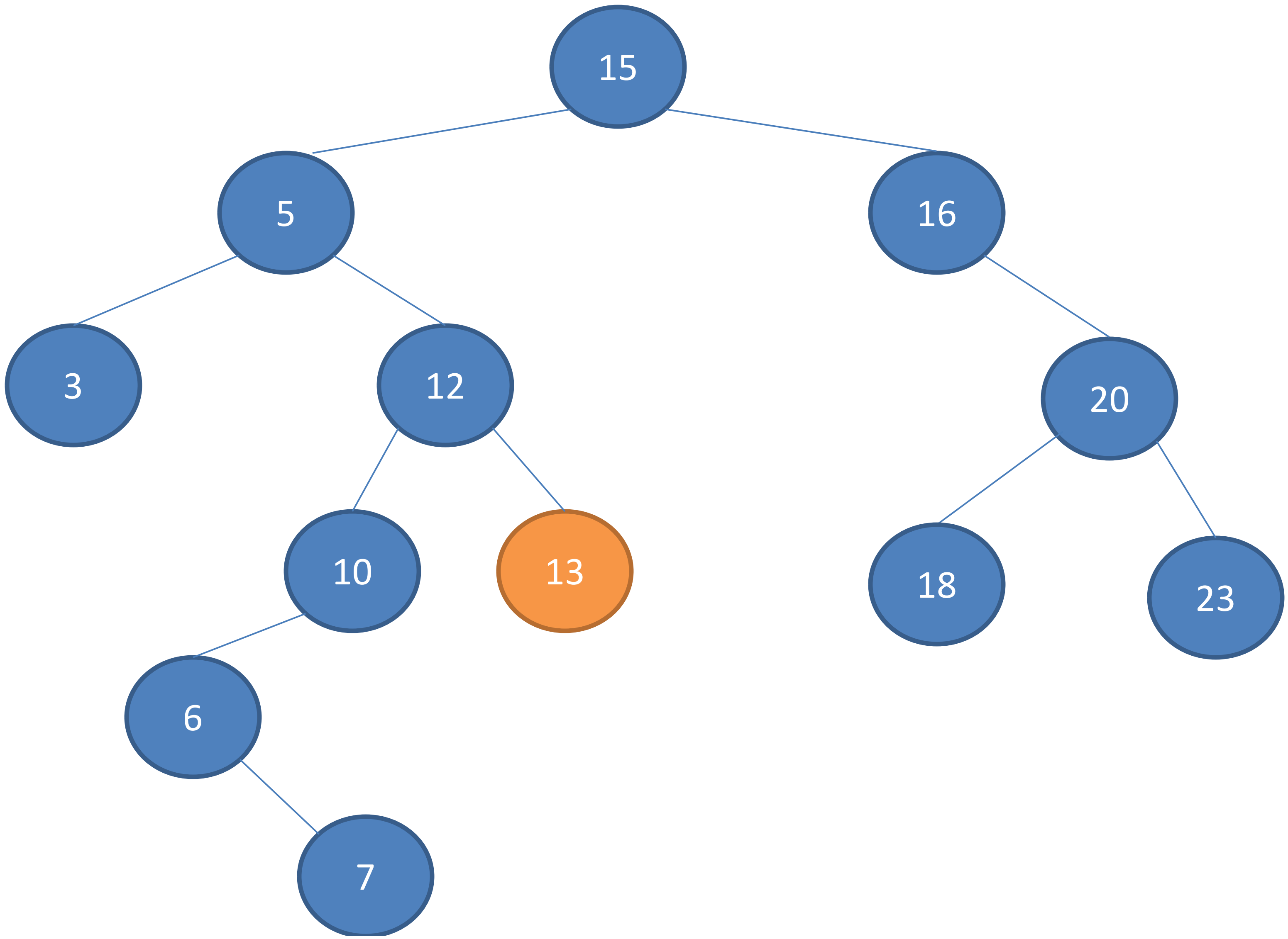


Recursion from a path downward from the root of the tree , so the running time is $\Theta(h)$

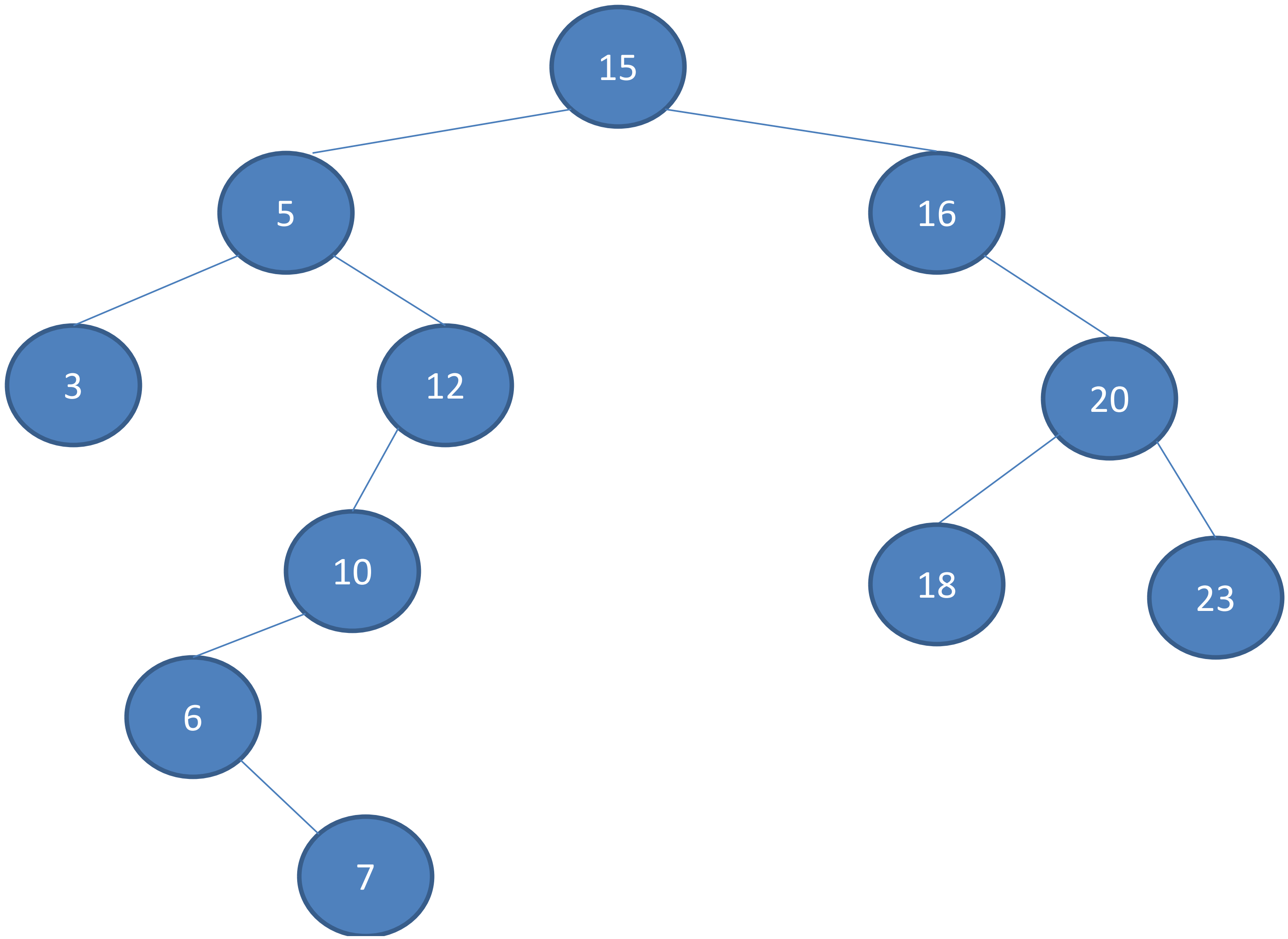
Tree-Delete (T,z)

```
if  left[z] = NIL or right[z] = NIL
            then y = z
            else y = Tree-Successor(z)
if  left[y] != NIL
            then x=left[y]
            else x = right[y]
If x != NIL
            then p[x] = p[y]
If p[y] = NIL
            then root[T] = x
            else if y = left[ p[y]]
                        then left[ p[y]] = x
                        else right[ p[y]] = x
if y != z
            then key[z] = key[y]
            copy y's satellite data into z
return y
```

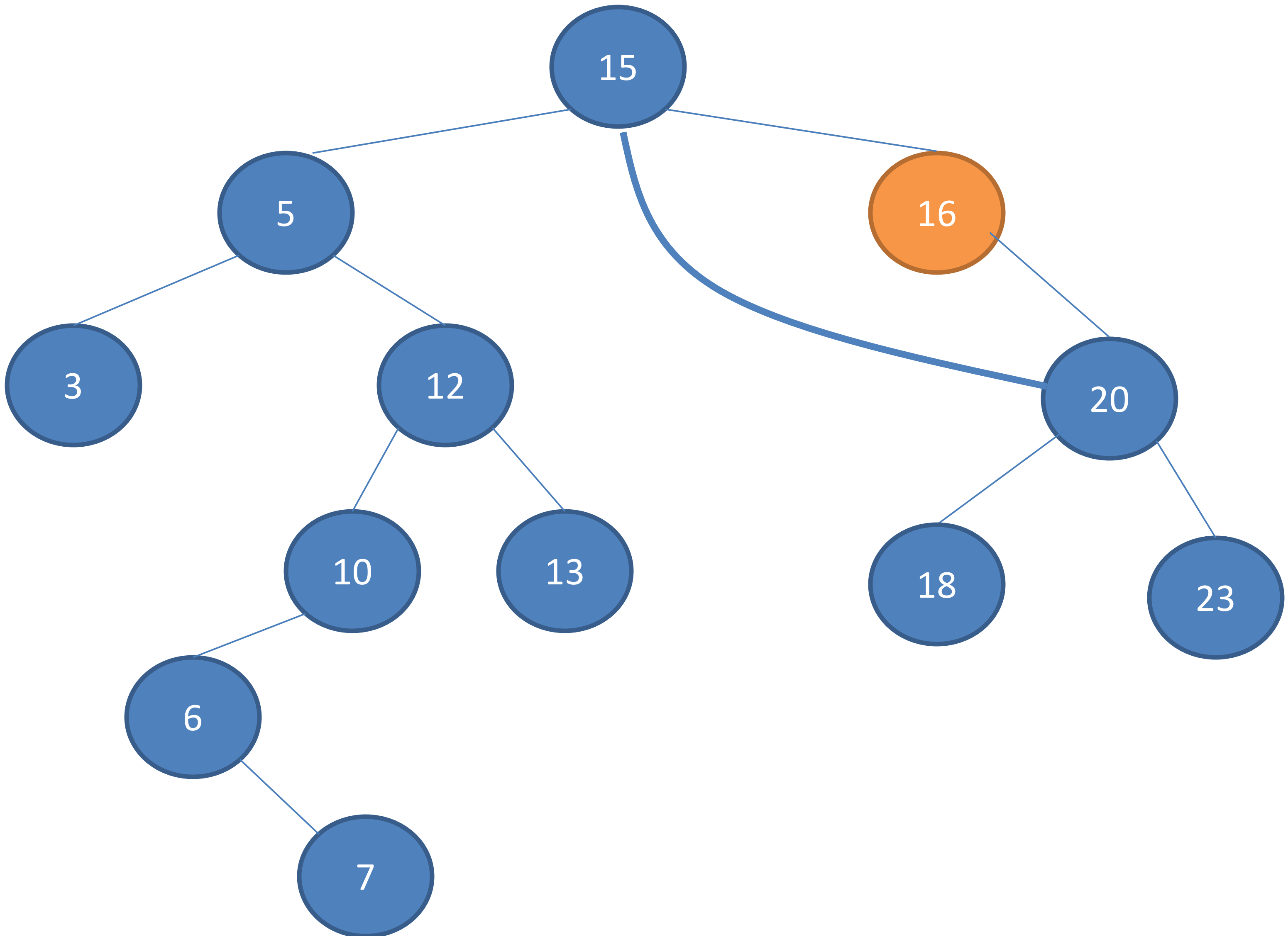
Example: Tree-Delete(T,13)



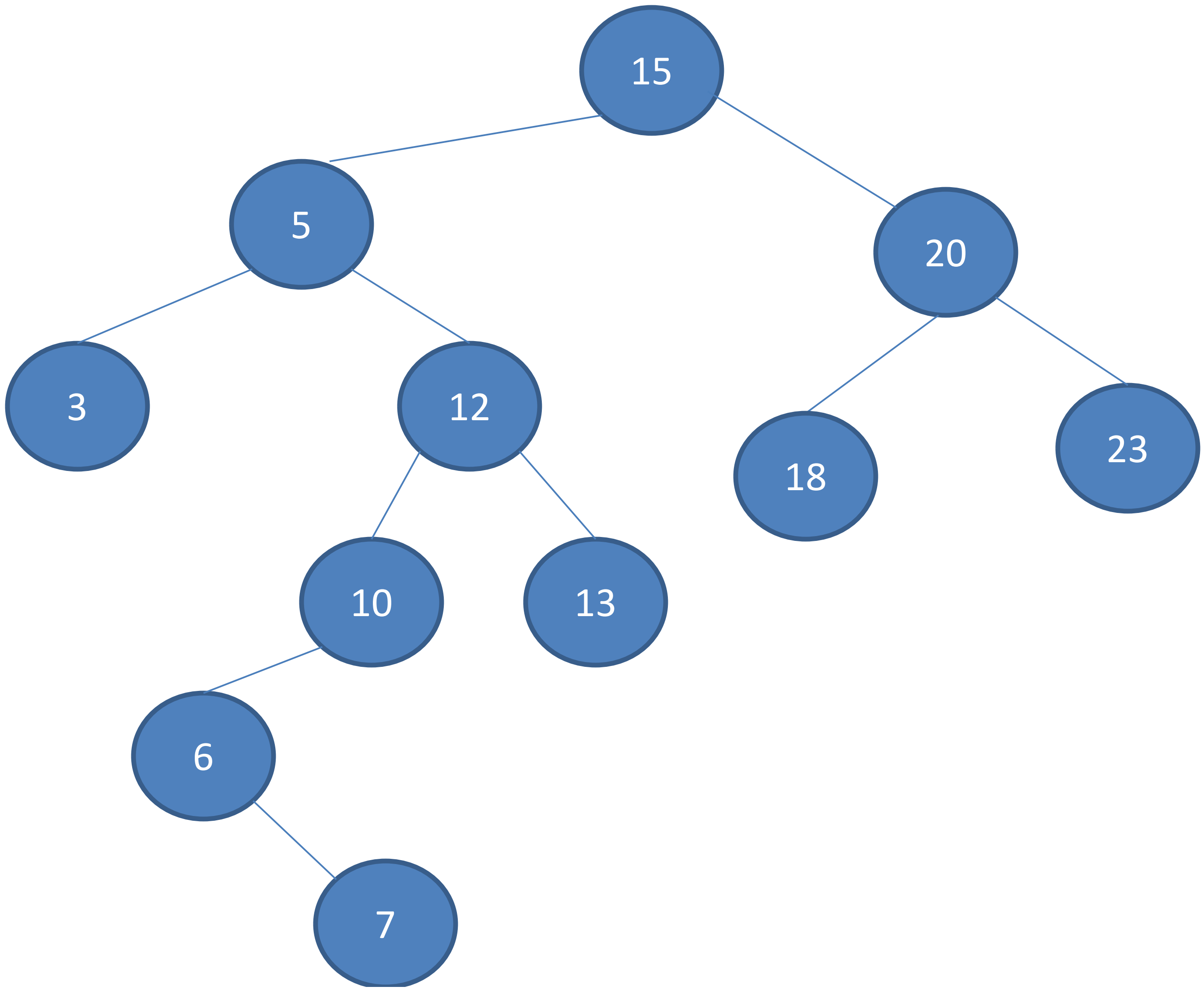
Example: Tree-Delete(T,13)



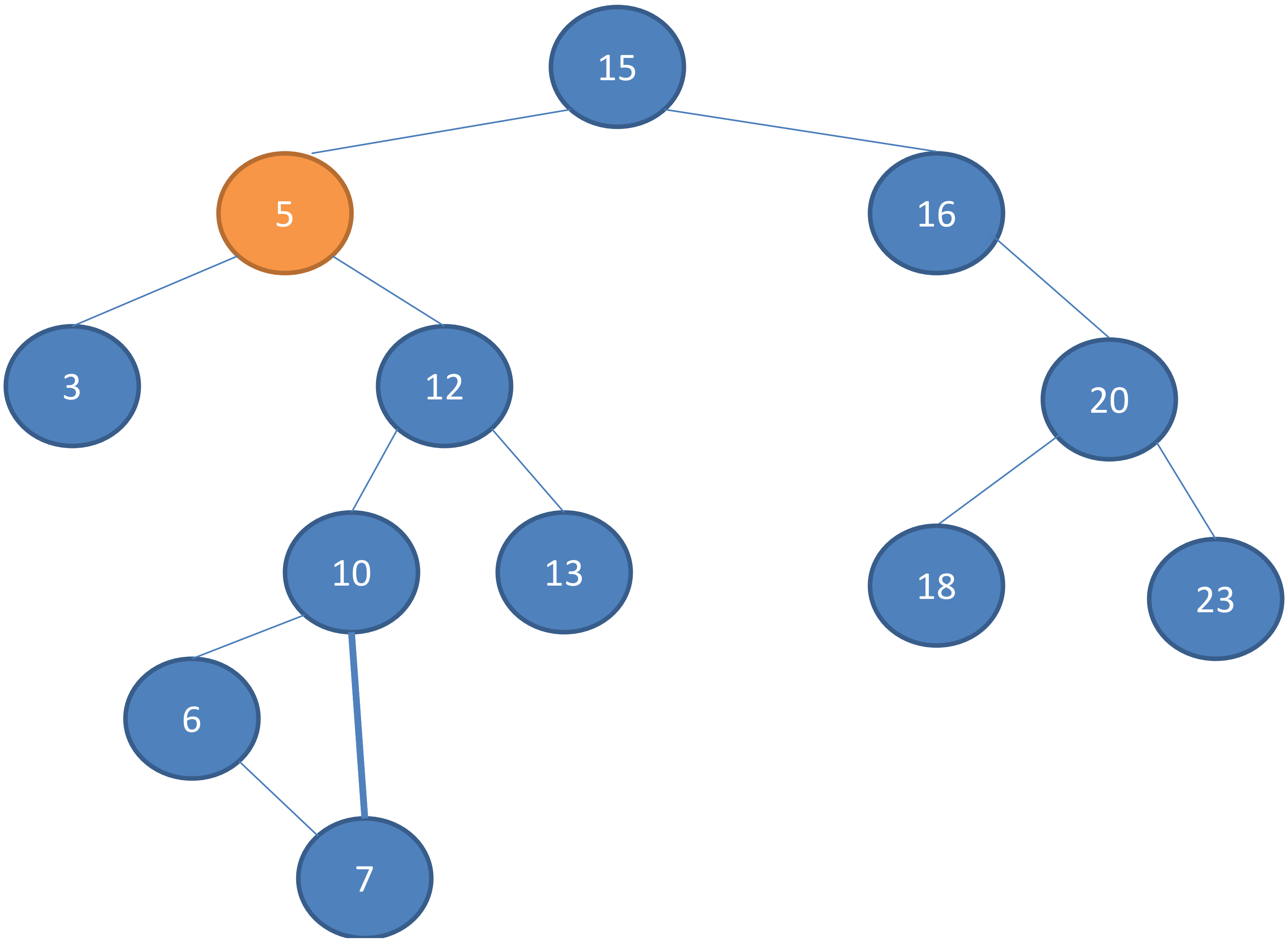
Example: Tree-Delete(T,16)



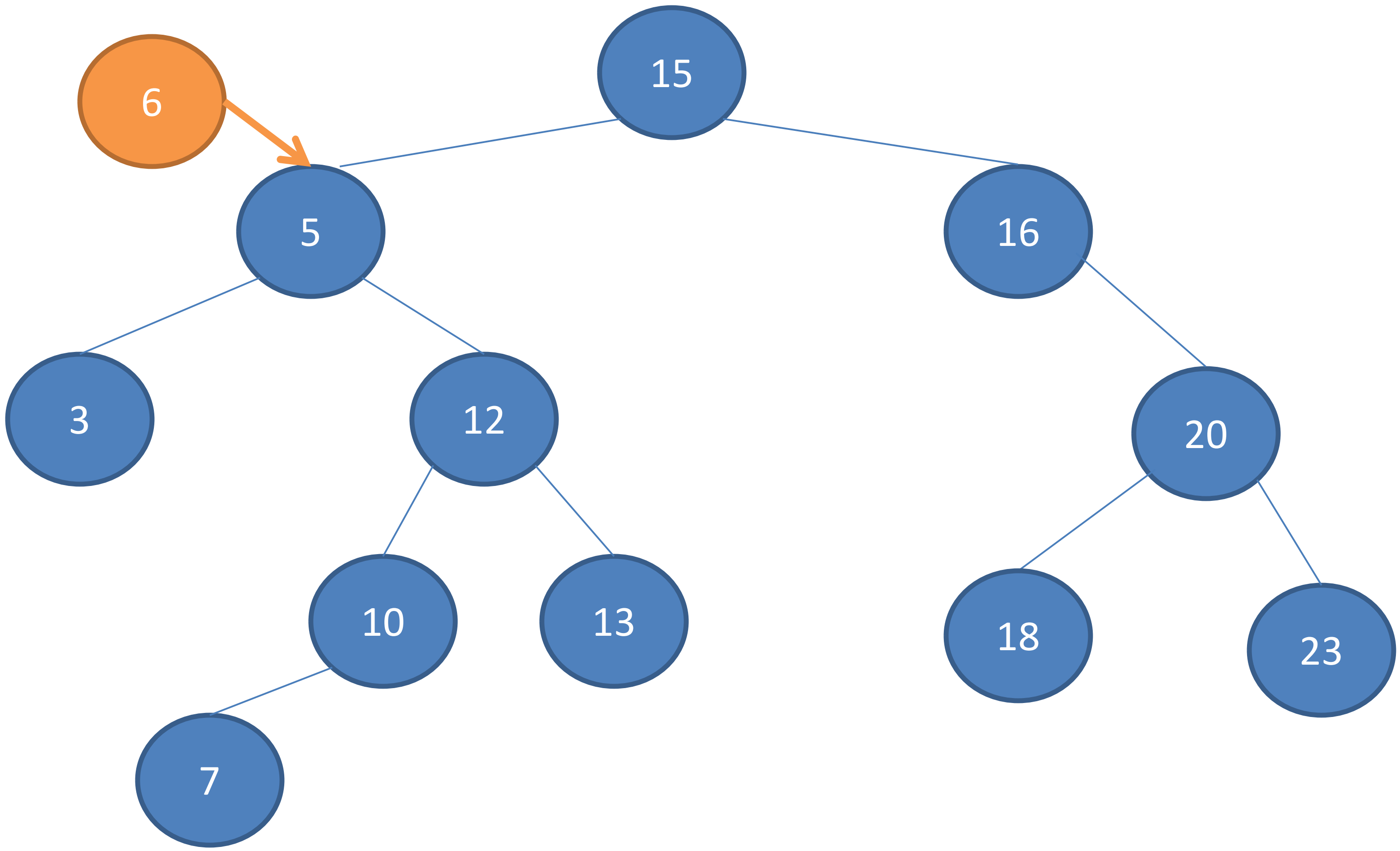
Example: Tree-Delete(T,16)



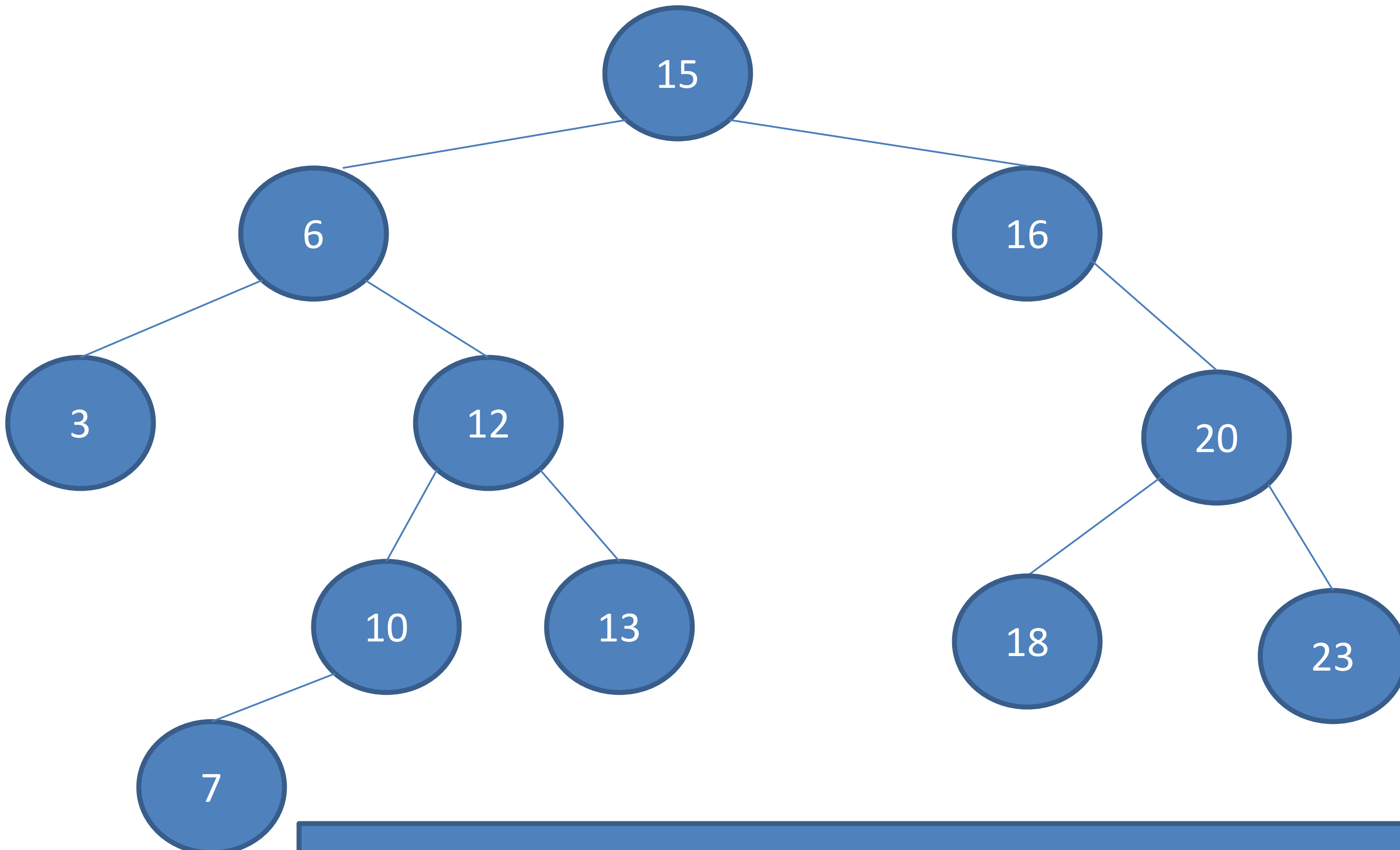
Example: Tree-Delete(T,5)



Example: Tree-Delete(T,5)



Example: Tree-Delete(T,5)



Recursion from a path downward from the root of the tree , so the running time is $\Theta(h)$