Ch15: AVL Tree

305233, 305234

Algorithm Analysis and Design

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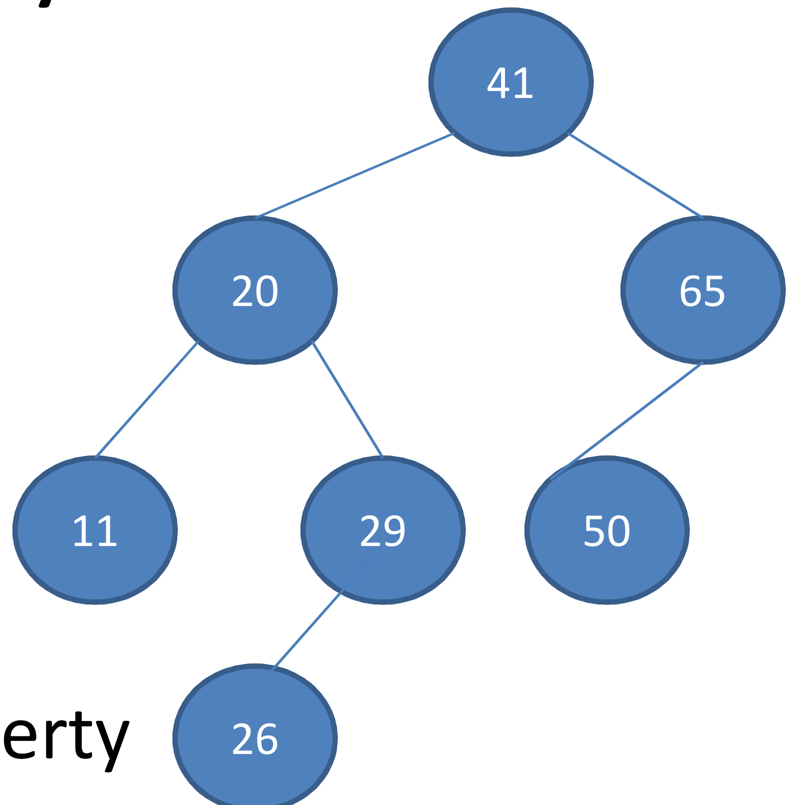
Review: Binary Search Tree

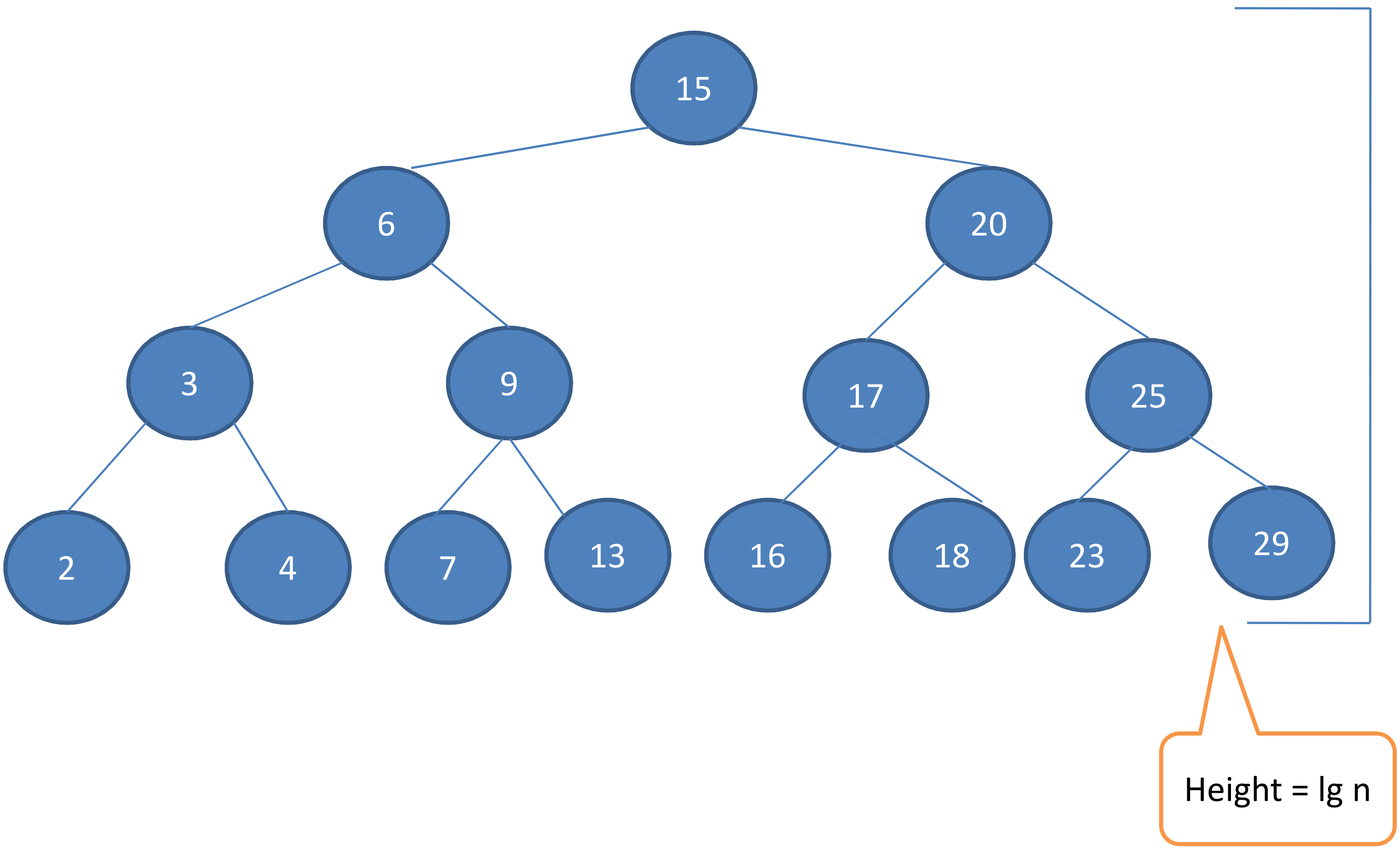


- Rooted binary tree
- Each node has
  - Key
  - Left pointer
  - Right pointer
  - Parent pointer
- Binary search tree property
  - Let x be a node in a binary search tree. If y is a node in the left subtree of x, then key[y] ≤ key[x]. If y is a node in the right subtree of x, then key[x] ≤ key[y].

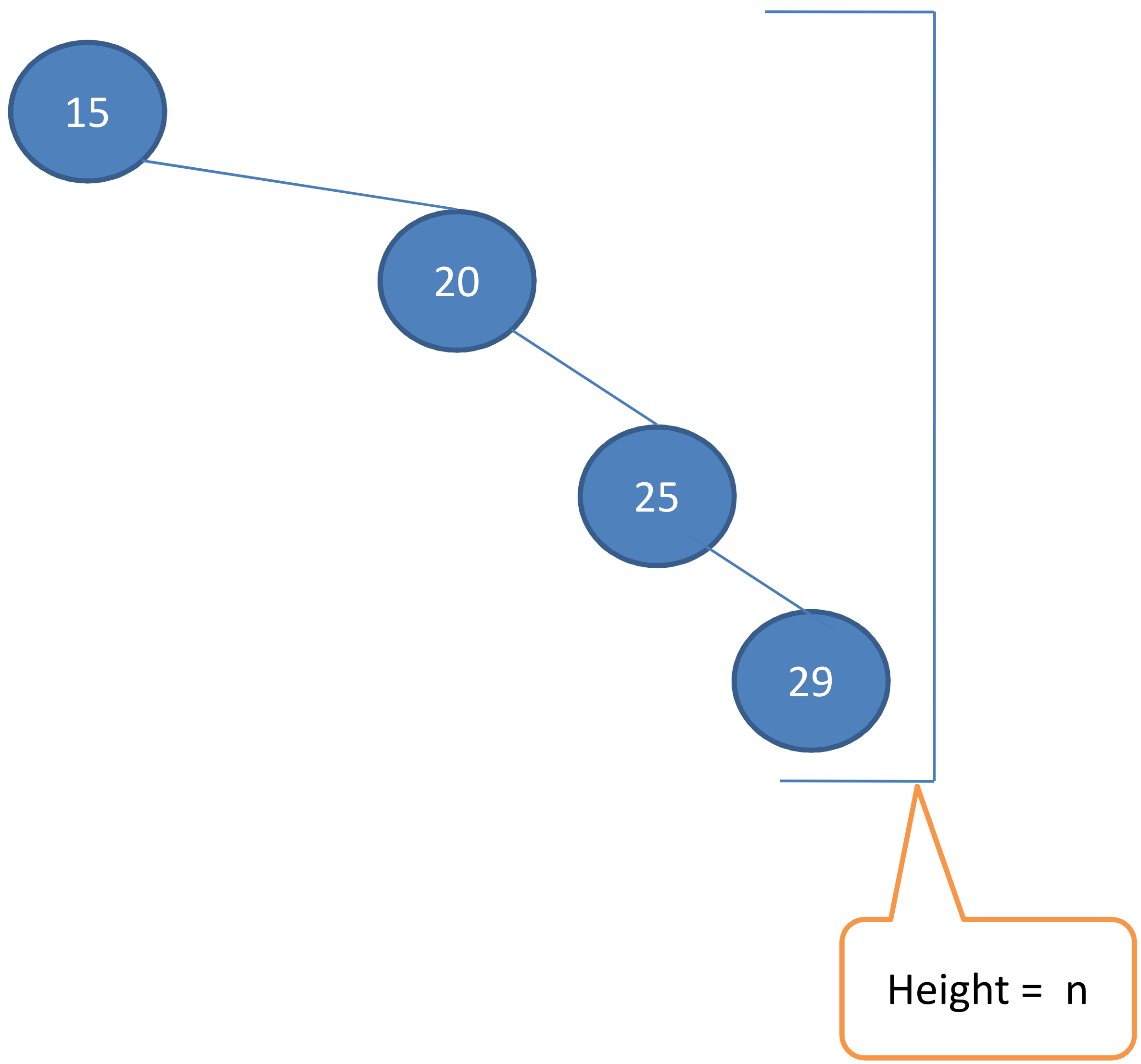
Review: Binary Search Tree

- Binary search tree supports
  - Insert
  - Delete
  - Minimum
  - Maximum
  - Successor
- In O(h) time where h is the height of a binary search tree

Height Balanced



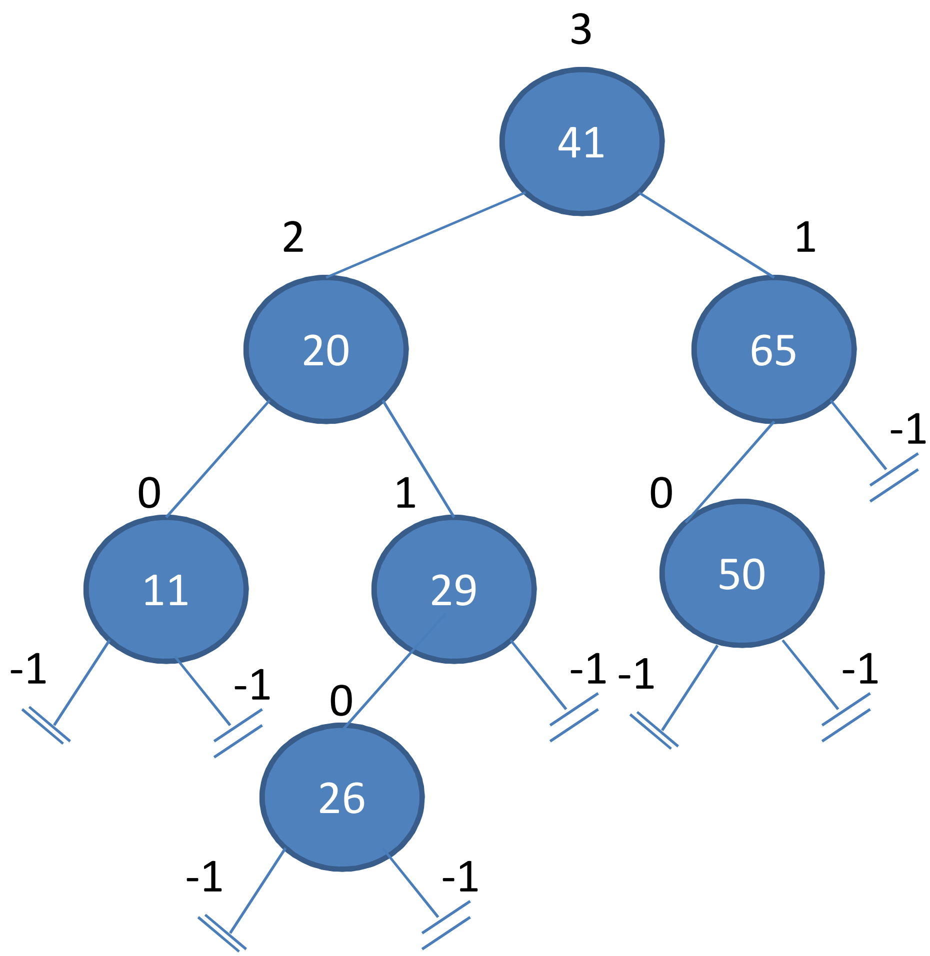
Height Unbalanced



Height

- **Height of a tree** is the length of longest path of the root down to a leaf.
- **Height of a node** is the length of longest path of it node down to a leaf.

Example: Height

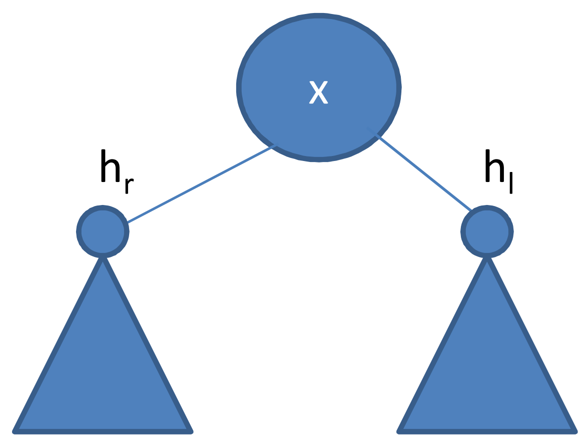


Height of node = max {height of left child , height of right child}

AVL Tree

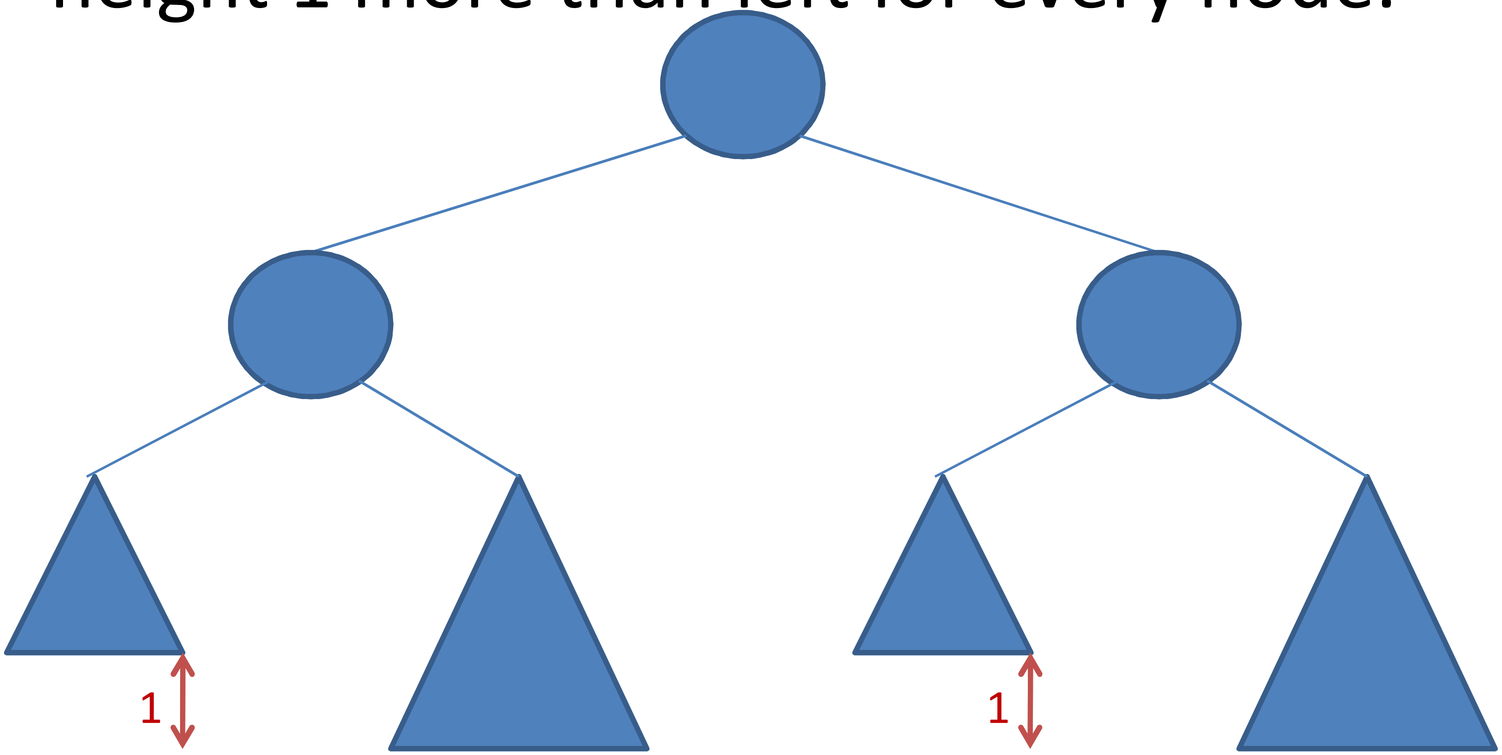
- An AVL tree is a binary search tree that is height balanced.
- For each node x, the heights of the left and right subtrees of x differ by at most 1.
- An AVL tree with n nodes has height O(lg n).

$$|h_r - h_r| \leq 1$$



AVL Tree

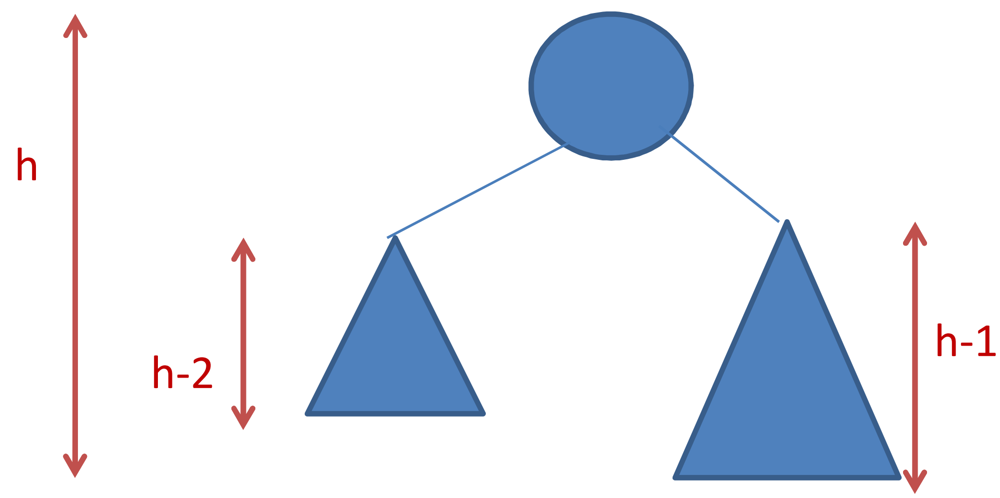
- AVL trees are balanced.
- The worst case is when the right subtree has height 1 more than left for every node.



Analyze the Height of AVL Tree

- Let N_h to be a minimum number of nodes in an AVL tree of height h.
- $N_h = 1 + N_{h-1} + N_{h-2}$
- $N_h > F_h$ where F_h is a Fibonacci function
- $N_h > 1.618^h / 5^{1/2}$
- Denote $N_h = n$
- $1.618^h / 5^{1/2} < n$
- $h < 1.440 \lg n$

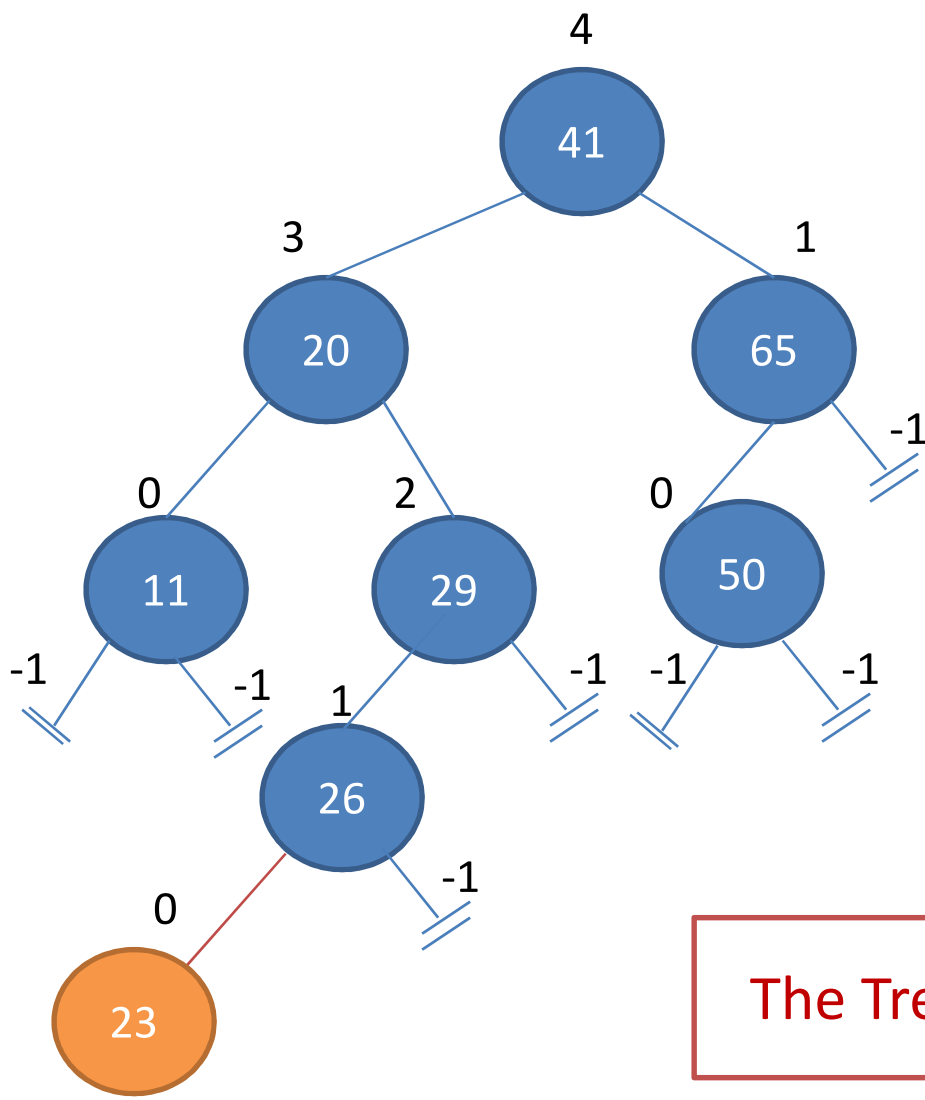
Height of AVL tree = lg n



AVL Insert

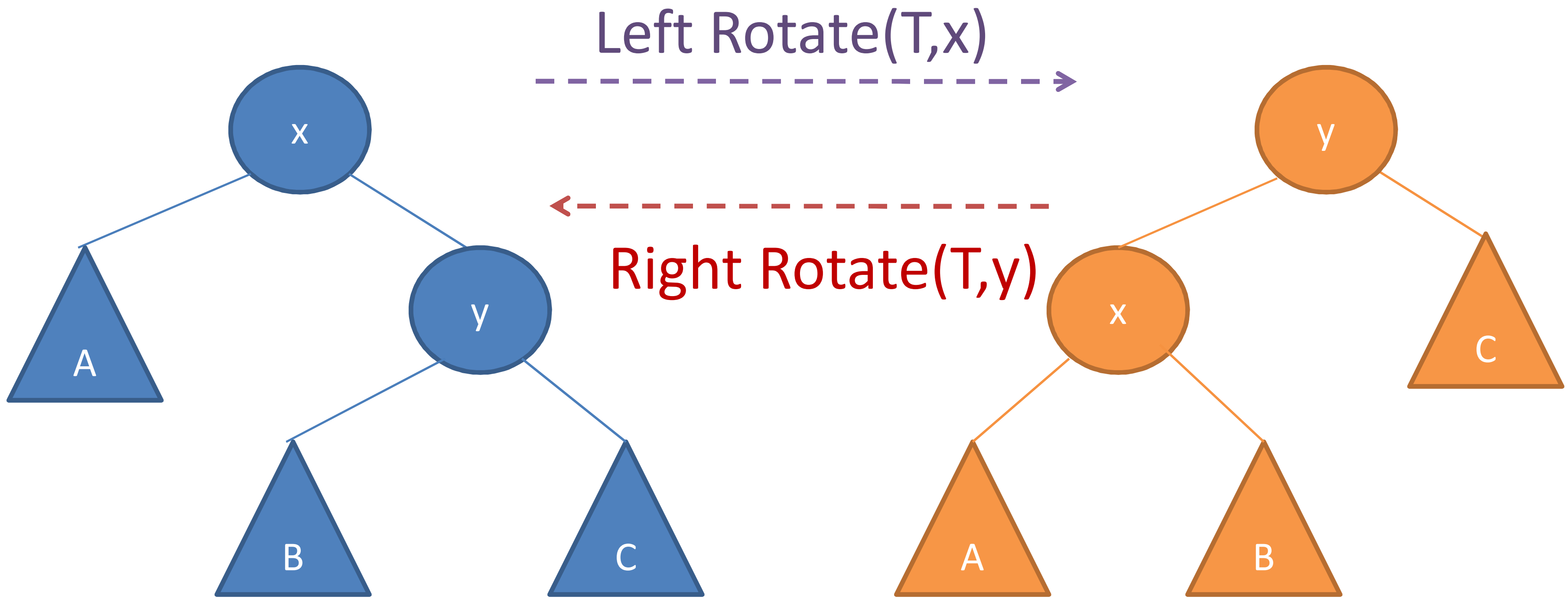
- Use a simple binary search tree insert
- Fix the AVL property using rotations

Example: AVL Insert(T,23)



The Tree is unbalanced !!!

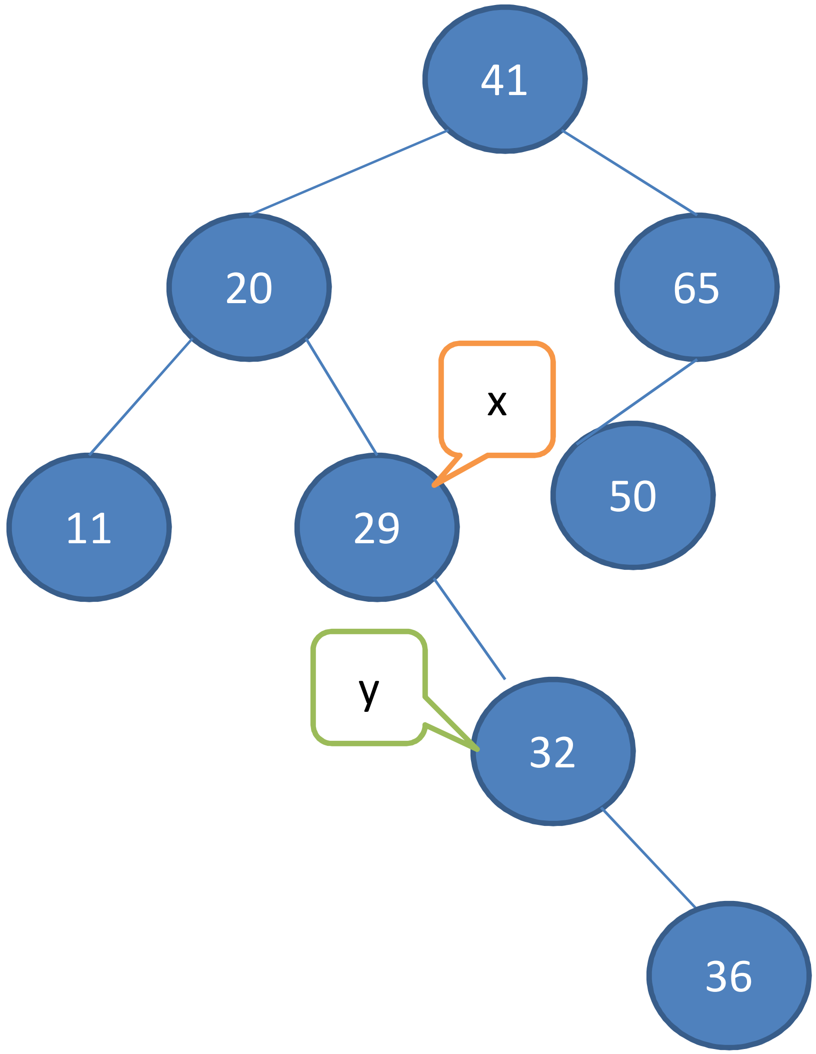
Rotations



Left-Rotate(T,x)

```
y = right[x]
right[x] = left[y]
if left[y] != nil[T]
        then p[left[y]] = x
p[y] = p[x]
if p[x]= nil[T]
        then root[T] = y
else if x = left[p[x]]
        then left[p[x]] = y
else right[p[x]] = y
left[y] =x
p[x] = y
```

Example: Left-Rotate(T,29)



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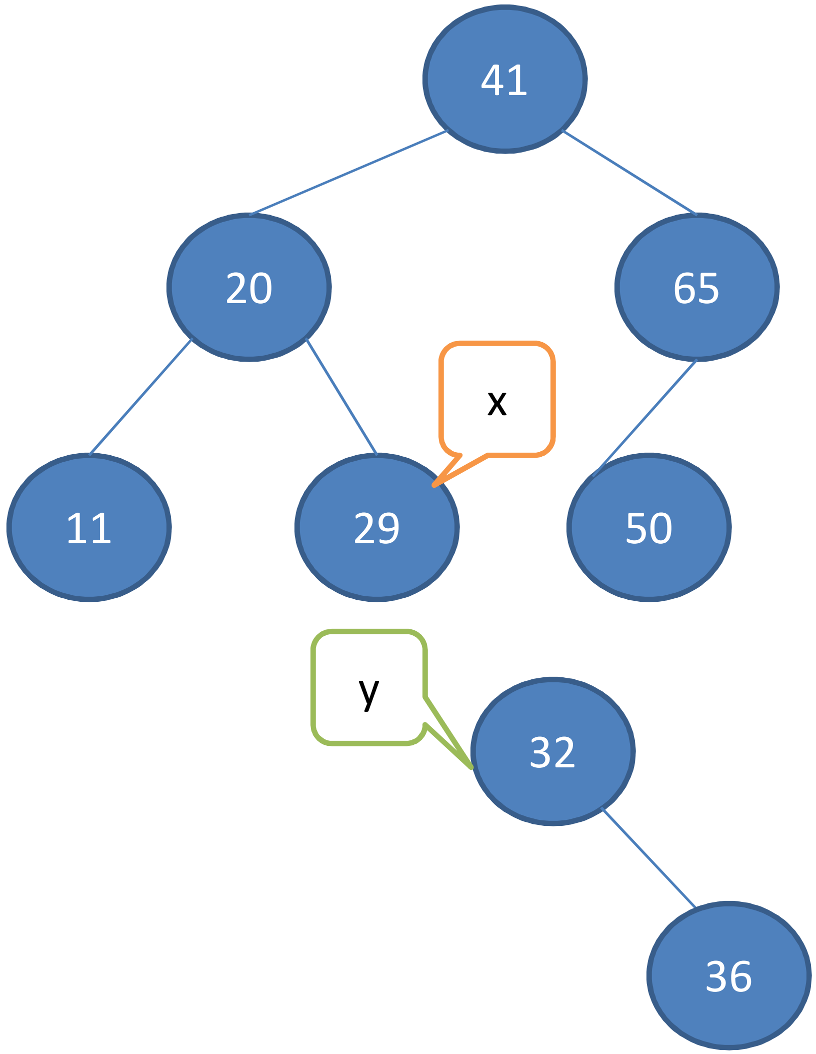

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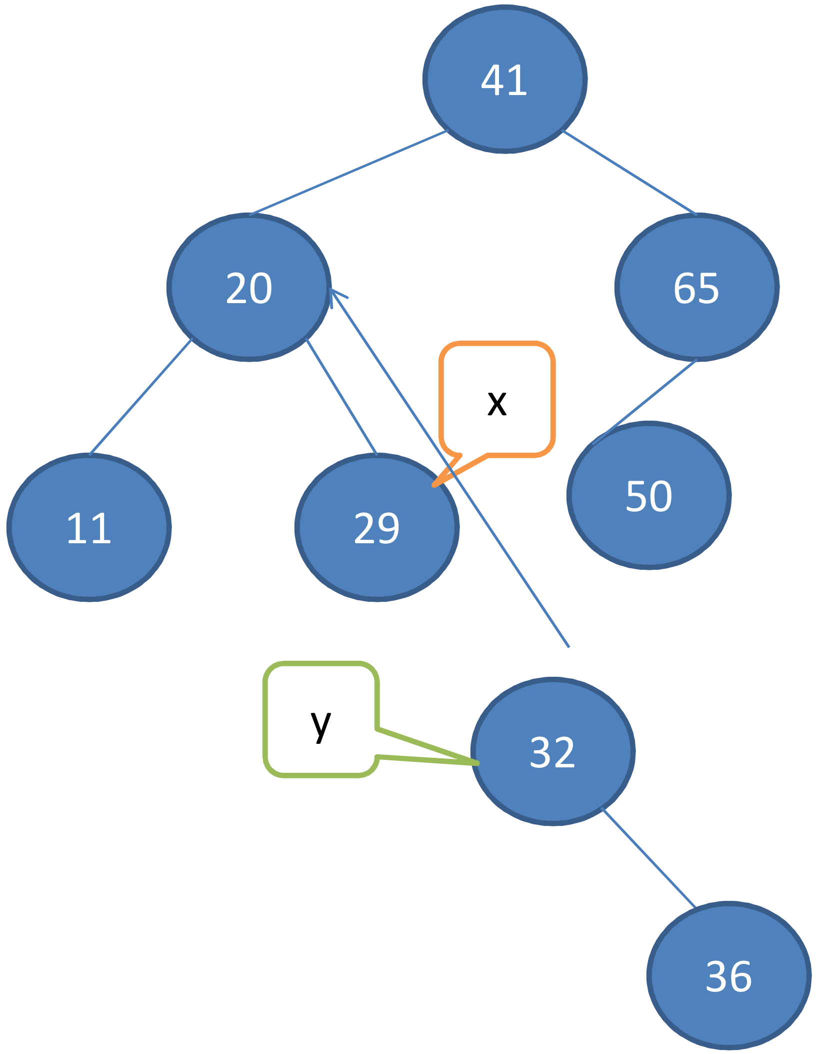

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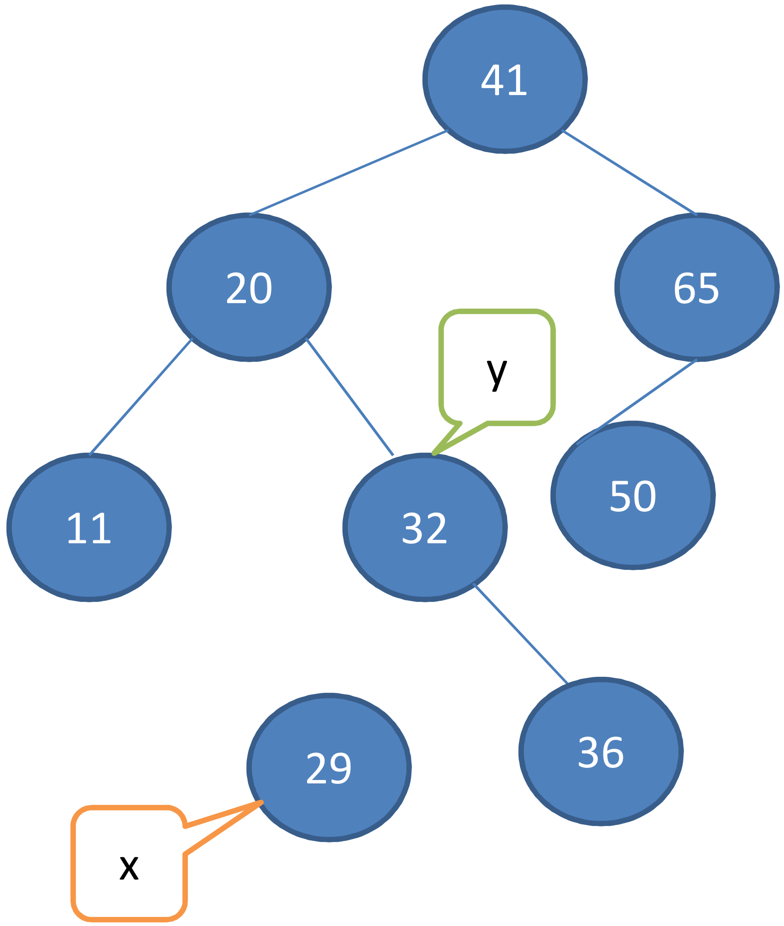

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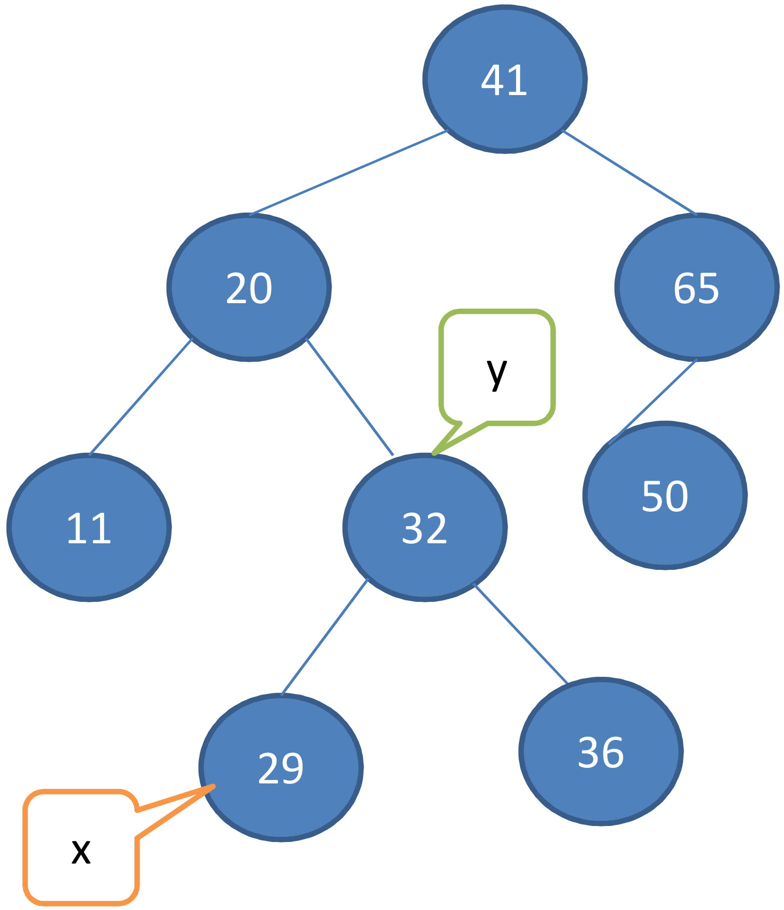

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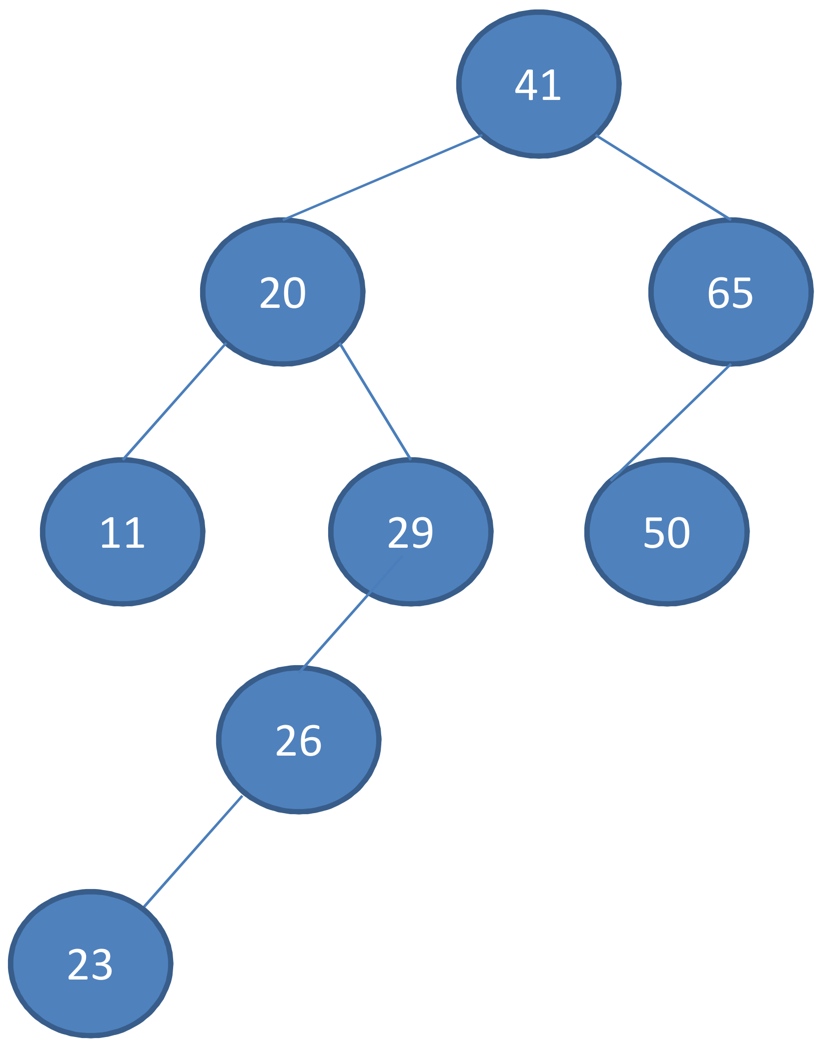

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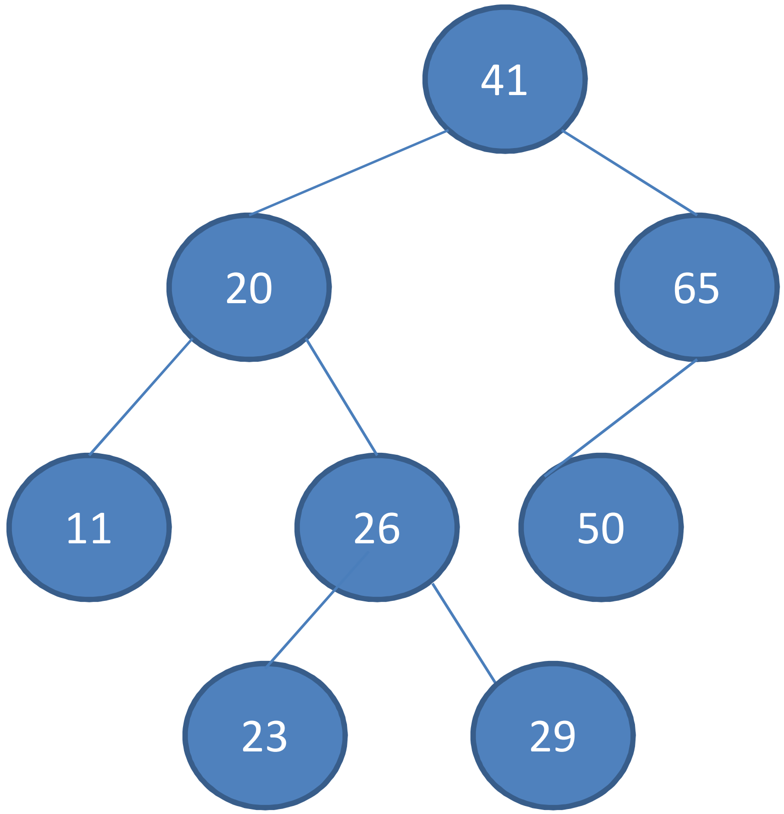
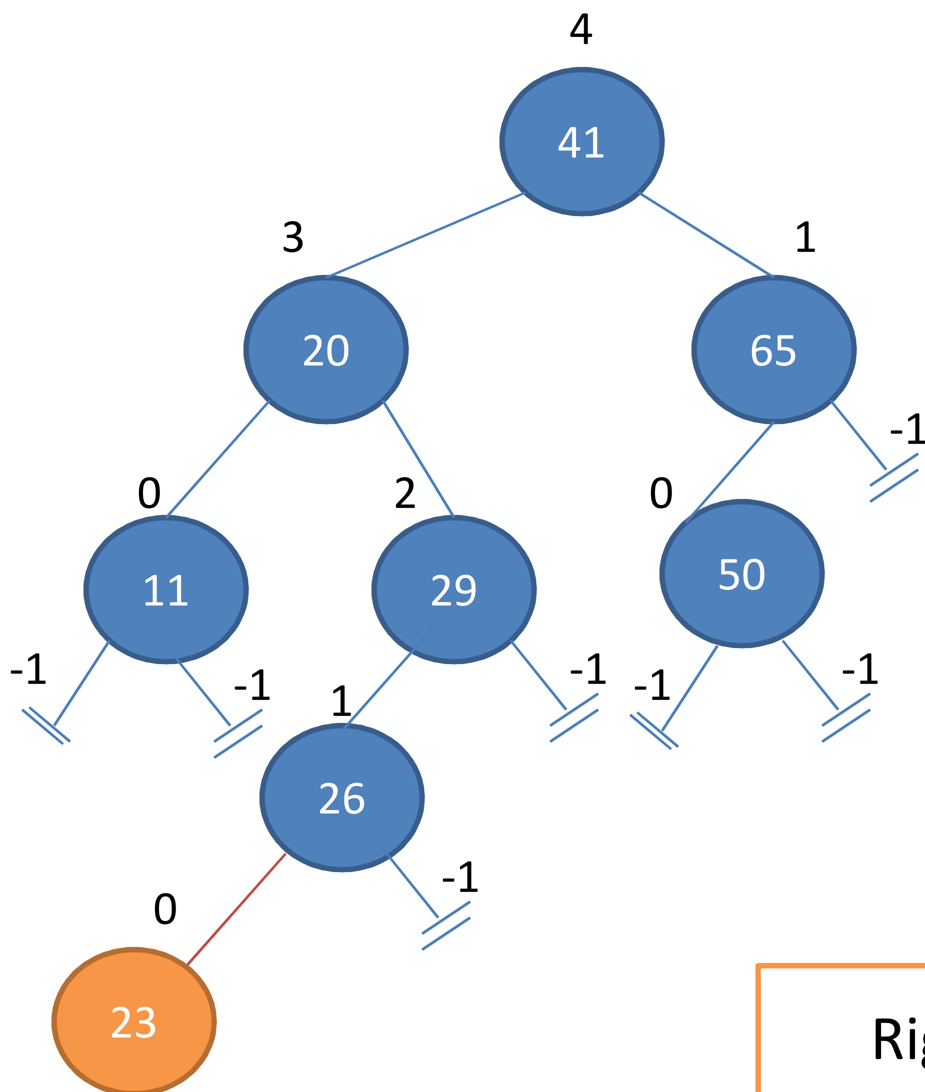
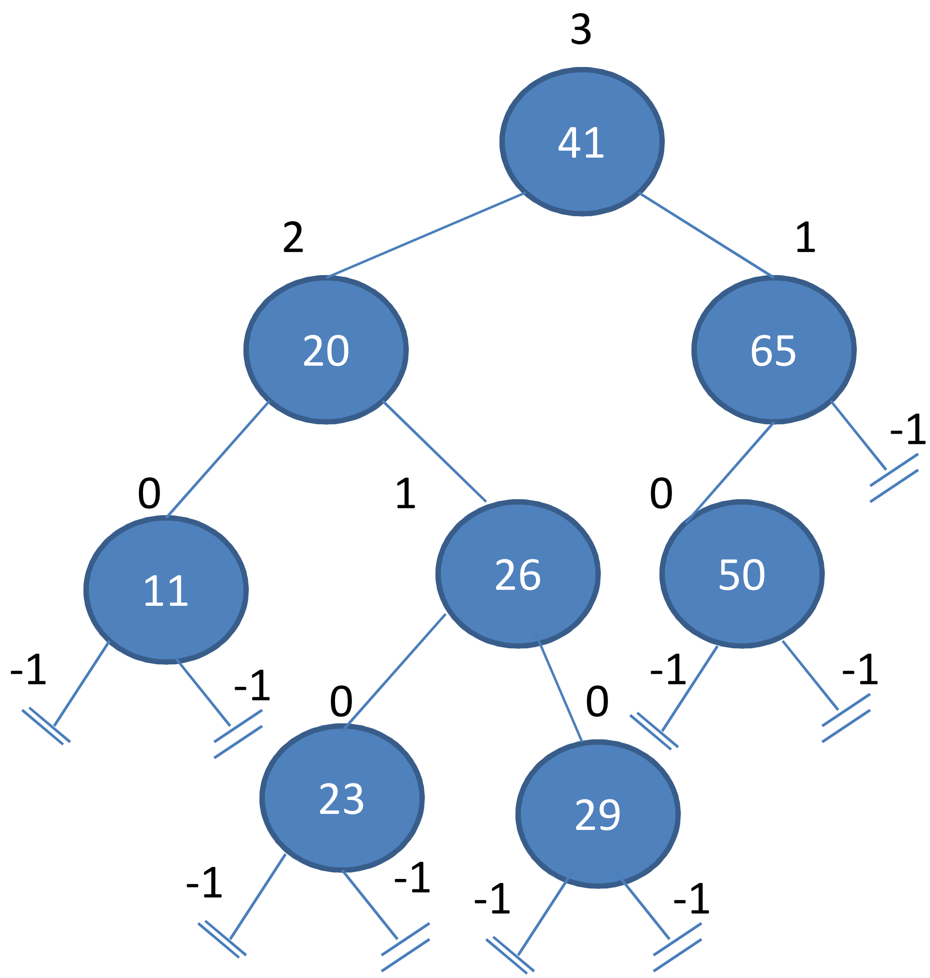


Example: AVL Insert(T,23)

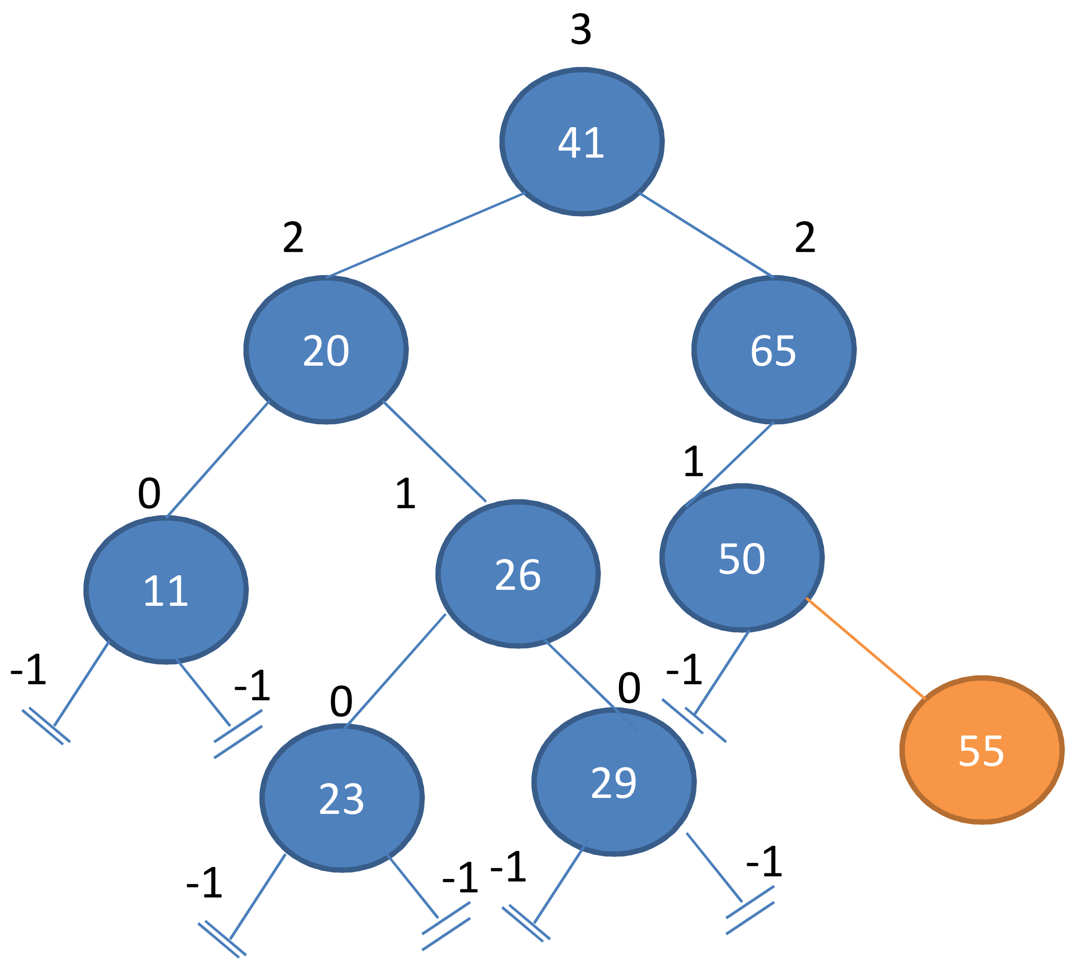


Right Rotate(T,29)

Example: AVL Insert(T,23)

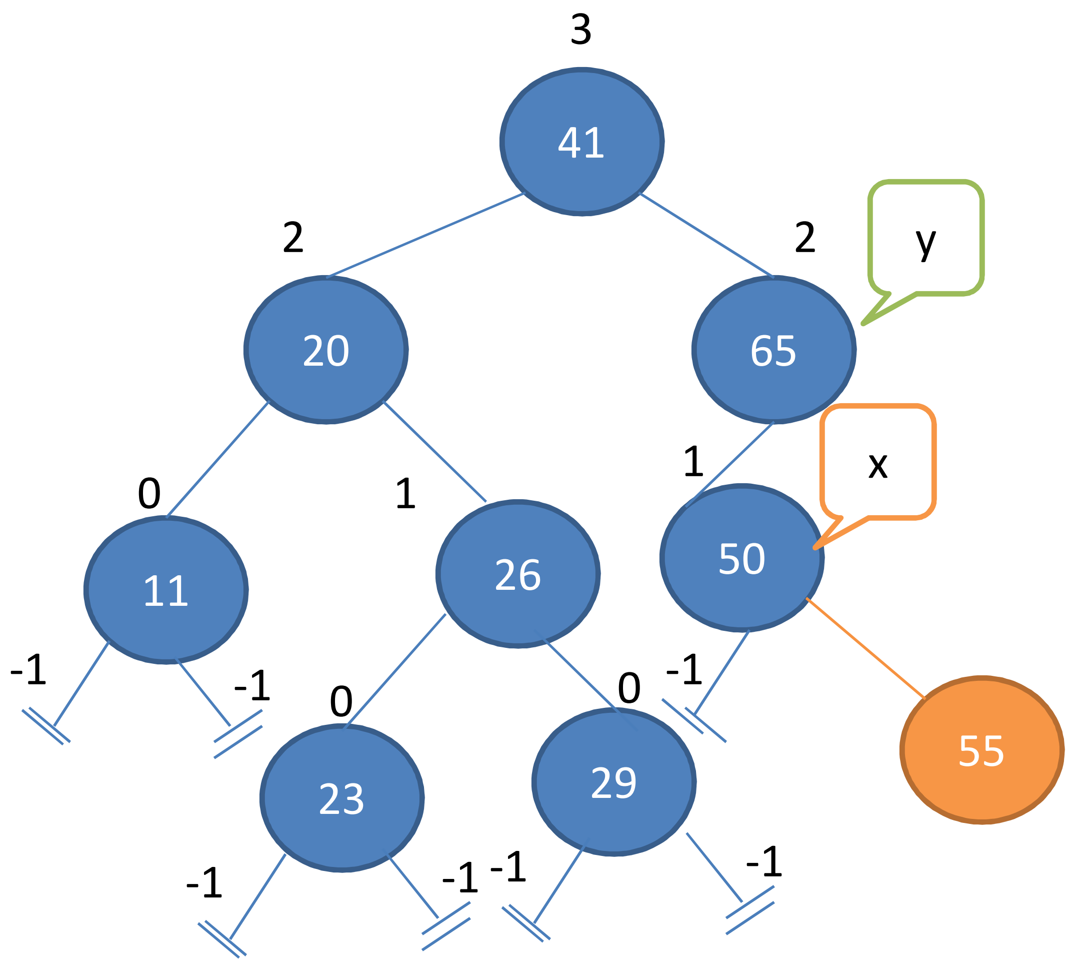


Example: AVL Insert(T,55)



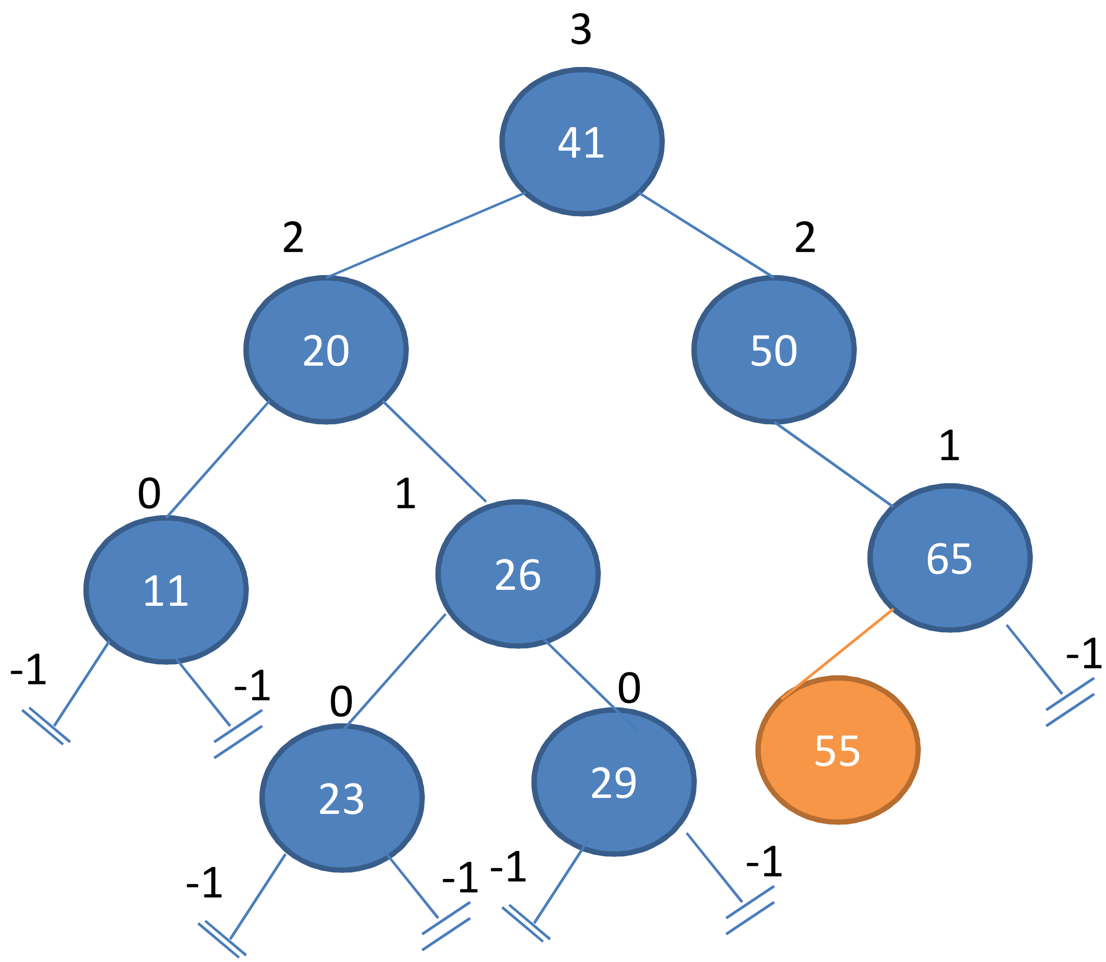
The Tree is unbalanced !!!

Example: AVL Insert(T,55)



Right Rotate(T,65)

Example: AVL Insert(T,55)



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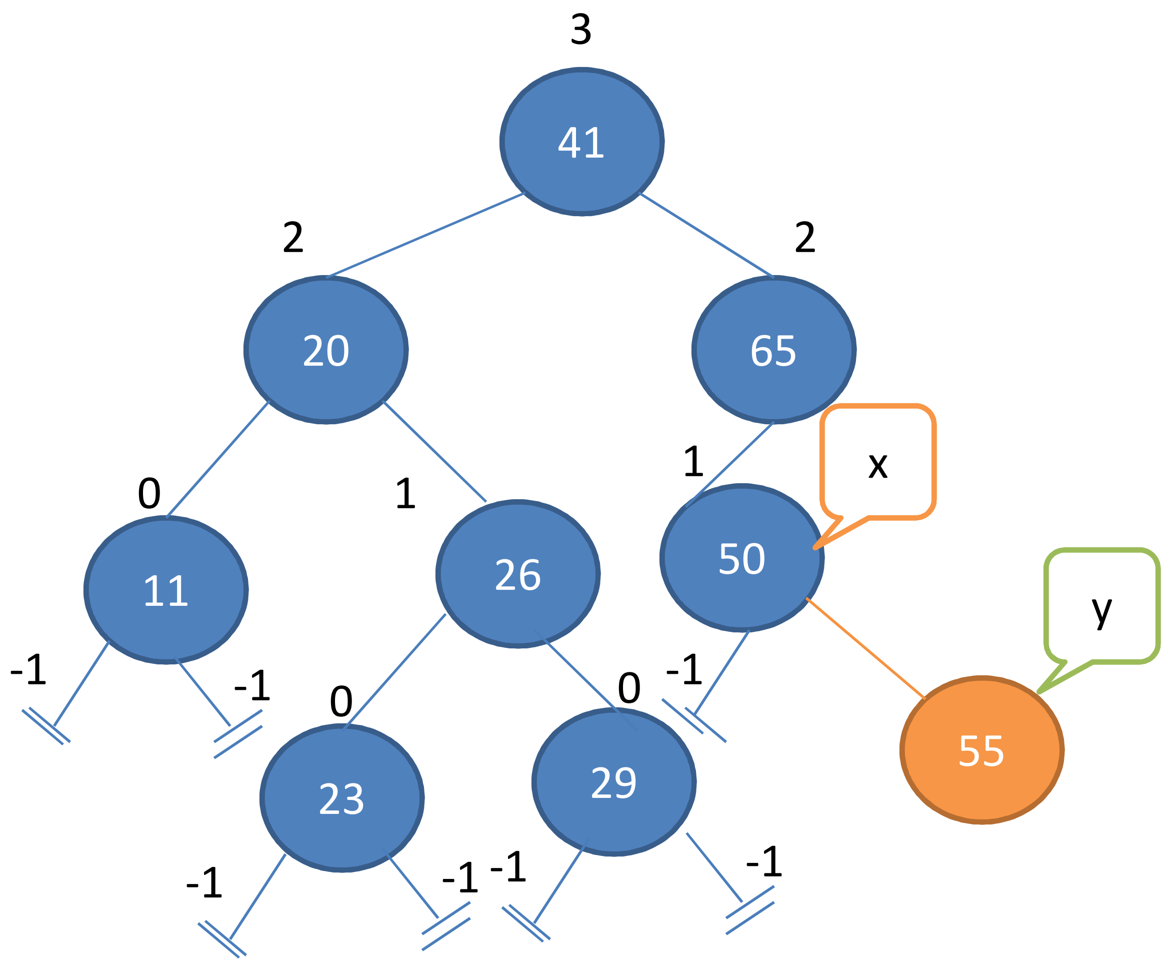
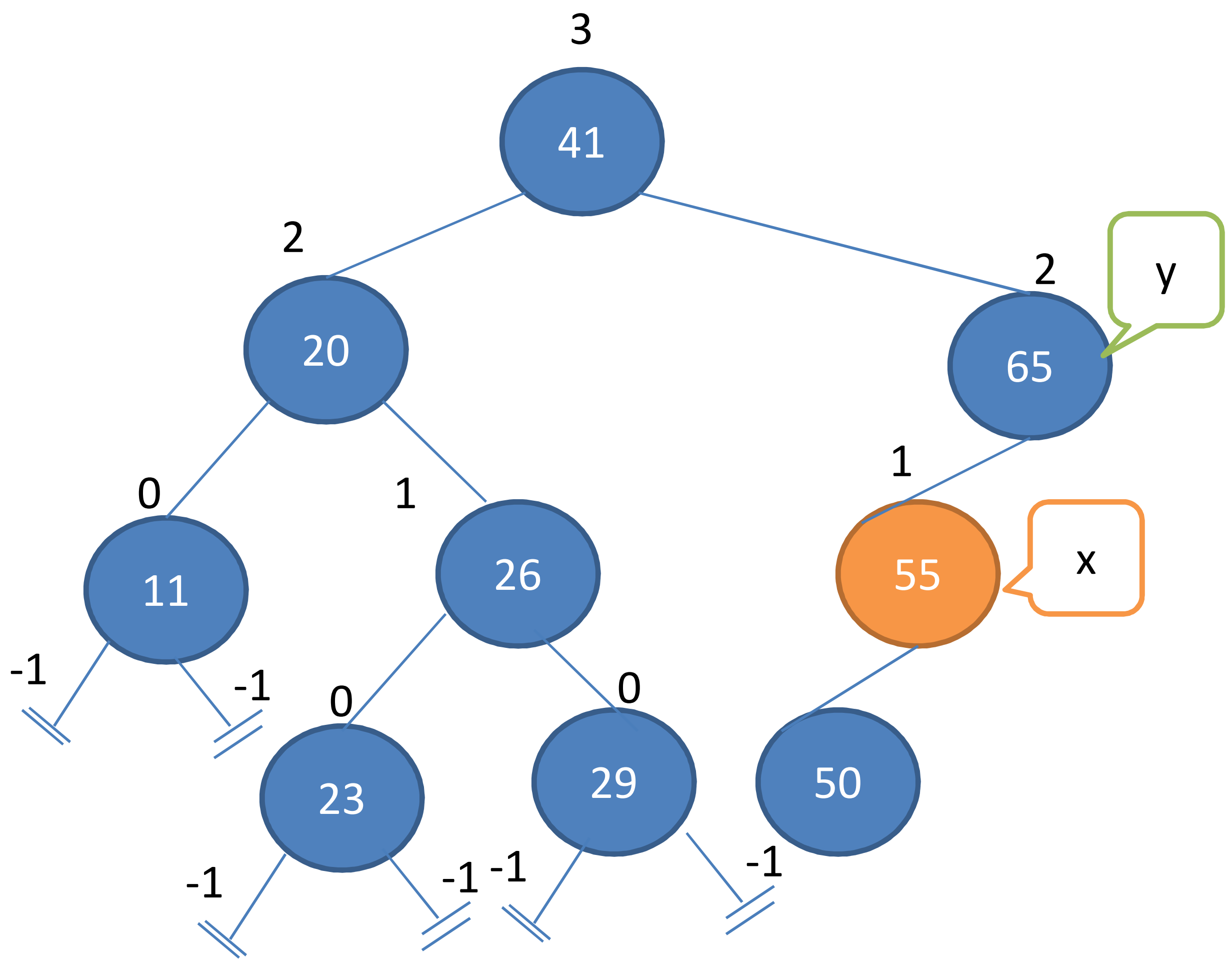
Example: AVL Insert(T,55)



Left-Rotate(T,50)

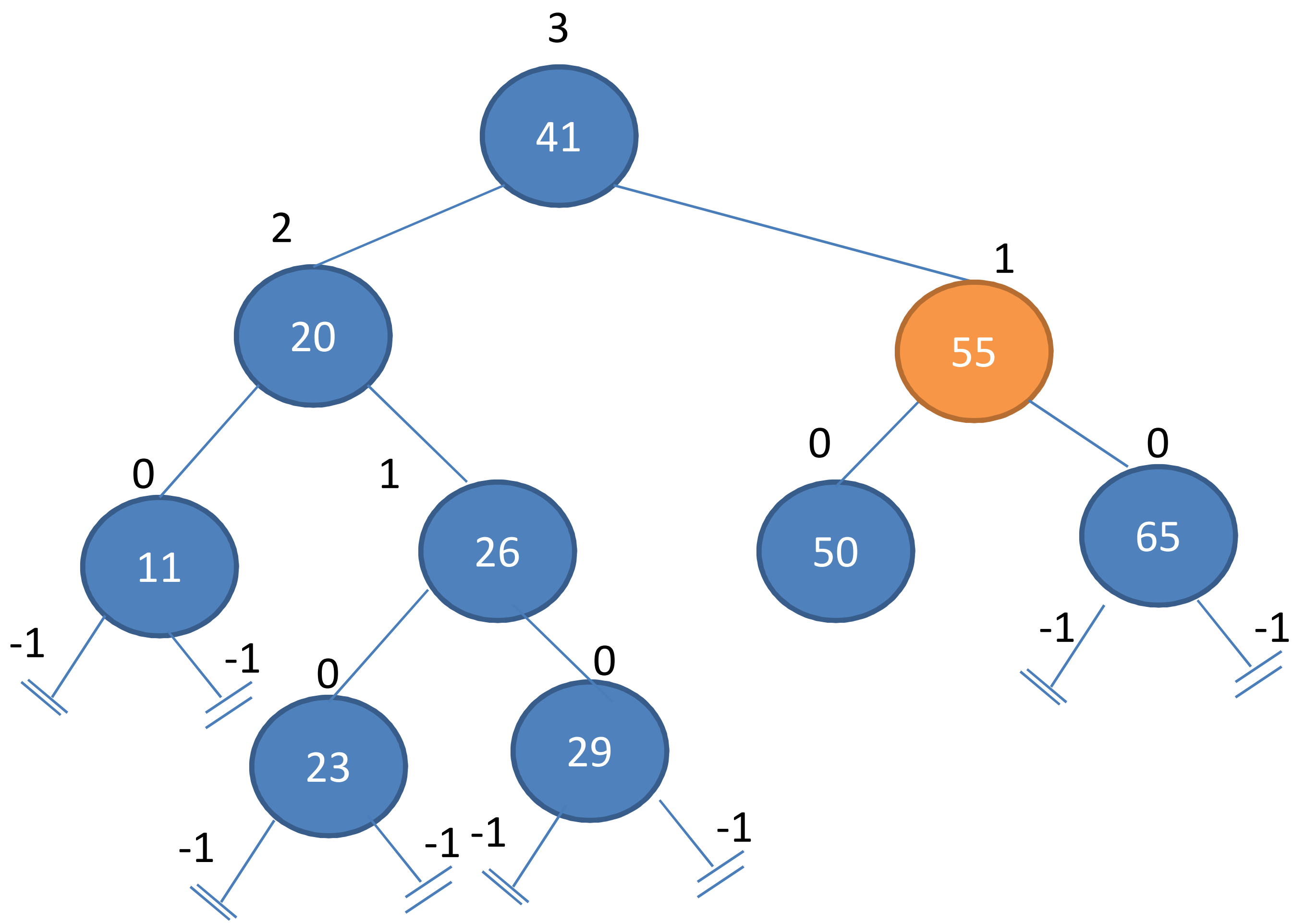
Example: AVL Insert(T,55)



Right-Rotate(T,65)

The Tree is unbalanced !!!

Example: AVL Insert(T,55)

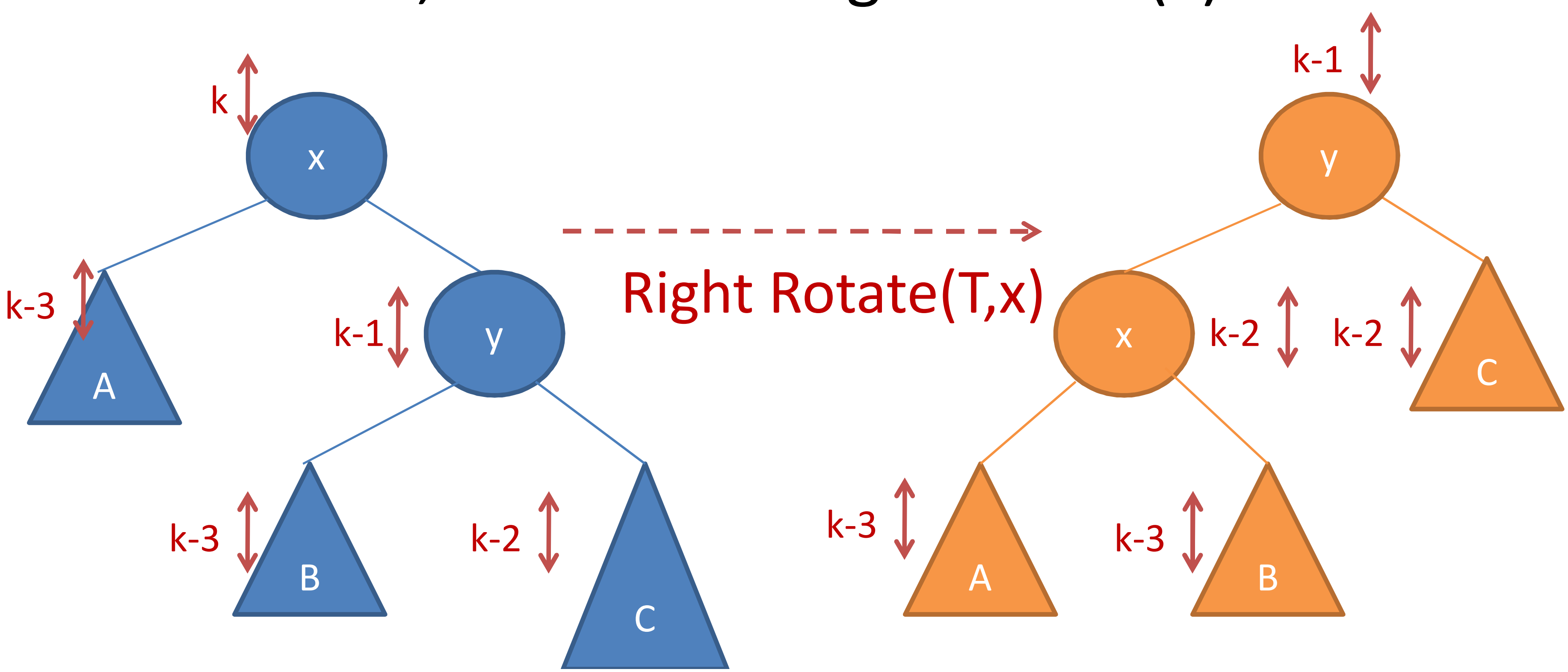


AVL Insert

- Use a simple binary search tree insert
- Fix the AVL property from changed node up
  - Suppose x is lowest node violating AVL property

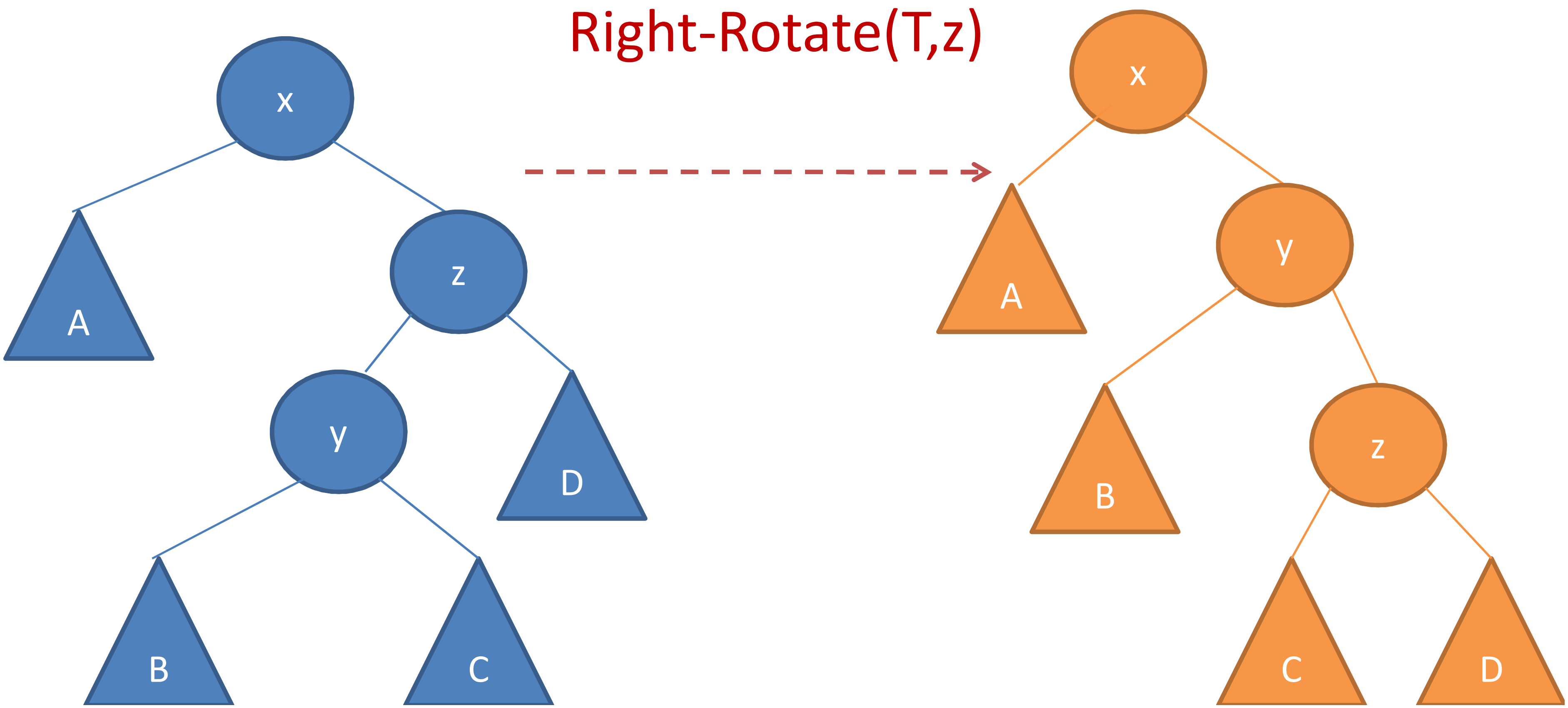
AVL Insert

- Assume the right child of x is higher
- If the right child of x is right-heavy or balanced, then we do right-rotate(x)



AVL Insert

- Assume the right child of x is higher
- Else



AVL Insert

- Assume the right child of x is higher
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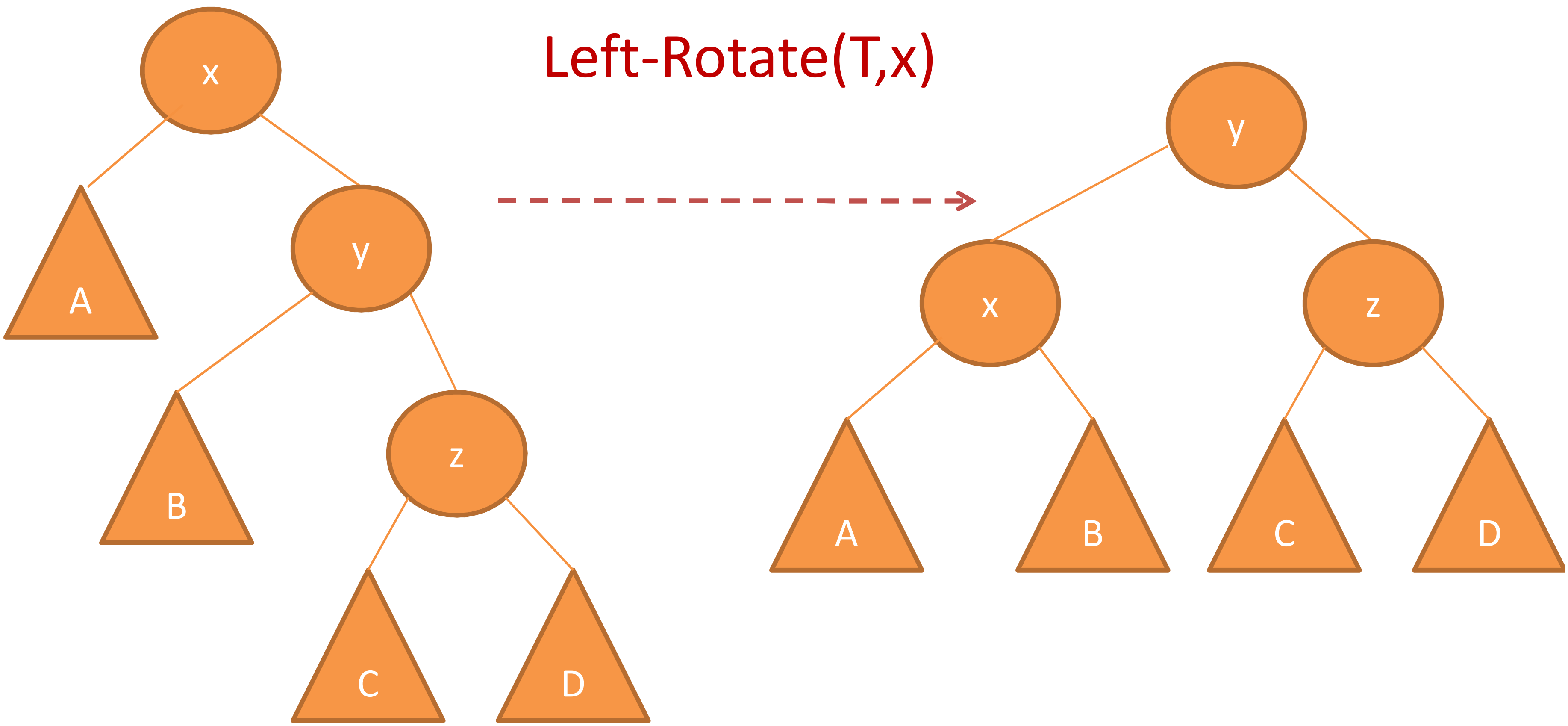
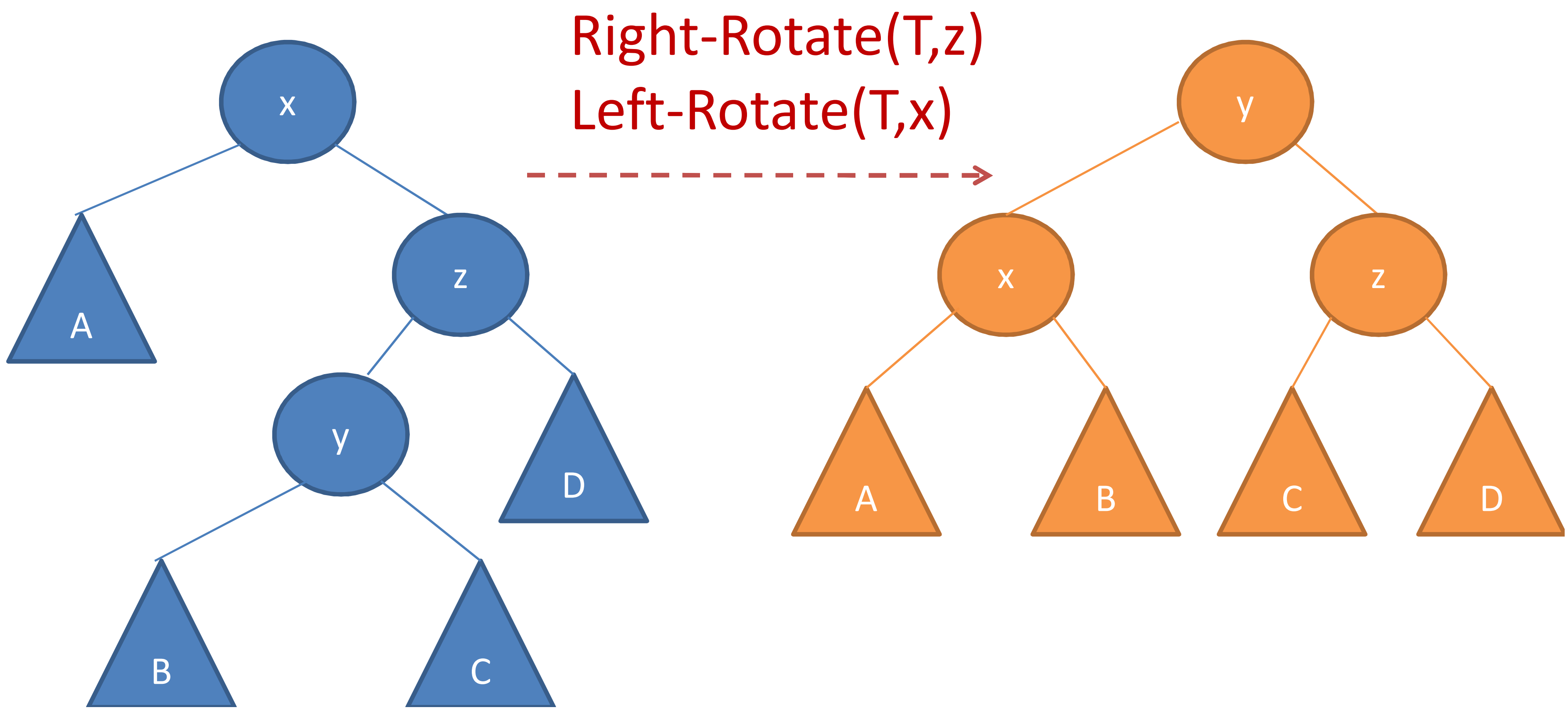


AVL Insert

- Assume the right child of x is higher
- Else



AVL Sort

- Insert n items
  - Take O(n lg n) time
- In-order tree traversal
  - Take O(n) time

Summary

- Abstract Data Types
- Insert , Delete
- Min
- Successor/ predecessor

Priority queue

Data Structure

- Heap or AVL
- Balanced BST

Practice: AVL Insert

4	7	5	10	23	65	73