Ch6: Analyzing Merge-Sort

305233, 305234

Algorithm Analysis and Design

Jiraporn Pooksook
Naresuan University

Analyze the Correctness of Algorithm

```
N1 = q – p + 1
N2 = r – q
Create arrays L[1..N1+1] and R[1…N2+1]
For i = 1 to N1
        do L[i]  = A[ p + i -1]
For j = 1 to N2
        do R[j] = A[q + j]
L[N1+1] =  ∞
R[N2+1] =  ∞
i = 1
j = 1
For k=p to r
        do if L[ i ] <= R[ j ]
                then A[k] = L[ i ]
                        i = i+1
                else A[k] = R[ j ]
                        j = j+1
```

Psudocode:
merge(A,p,q,r)

Loop invariants with merge(A,p,q,r)

theoretically

loop invariant =
before running loop k, all elements in A[p … k – 1] contains k-p smallest number of L[1..N1+1] , R[1…N2+1]are in sorted order and, L[i] , R[j] are the smallest number of their arrays that have never been copied into A.

Initialization:
Before running loop k , k = p then all elements in = A[p…p-1] is empty.
Hence A[p…k-1] are sorted.
And i =j =1 hence, L[i] and R[j] are the smallest number. (True!!)

Maintenance:
If before running loop k , all elements in = A[p…k-1] contains k-p smallest number of L[1..N1+1], R[1…N2+1] are in sorted order.
then after running loop k, if L[i] <= R[j] then L[i] is the smallest and will be copied to position k
and A[p…k-1] are sorted.
Hence A[p…k] will contains k – p +1 smallest number.
if L[i] > R[j] then R[j] is the smallest and will be copied to position k
and A[p…k-1] are sorted.
Hence A[p…k] will contains k – p +1 smallest number.
Hence before running loop k+1, A[p…k] are sorted. (True!!)

Loop invariants with merge(A,p,q,r)

theoretically

loop invariant =
before running loop k, all elements in A[p … k – 1] contains k-p smallest number of L[1..N1+1] , R[1…N2+1]are in sorted order and, L[i] , R[j] are the smallest number of their arrays that have never been copied into A.

Maintenance:
If before running loop k , all elements in = A[p…k-1] contains k-p smallest number of L[1..N1+1], R[1…N2+1] are in sorted order.
then after running loop k, if L[i] <= R[j] then L[i] is the smallest and will be copied to position k and A[p…k-1] are sorted.
Hence A[p…k] will contains k – p +1 smallest number.
if L[i] > R[j] then R[j] is the smallest and will be copied to position k and A[p…k-1] are sorted.
Hence A[p…k] will contains k – p +1 smallest number.
Hence before running loop k+1, A[p…k] are sorted. (True!!)

Termination: at starting of loop k+1, A[p…k] contain k+1 – p smallest number of L[1..N1+1], R[1…N2+1] are in sorted order. (True!!)

THE RUNNING TIME OF ALGORITHM

Analyze the Running time of merge-sort

```
If p < r
        then q = L(p+r)/2
                merge-sort(A, p, q)
                merge-sort(A, q+1, r)
                merge(A, p, q, r)
```

Divide
$T1 = t_1 * 1$

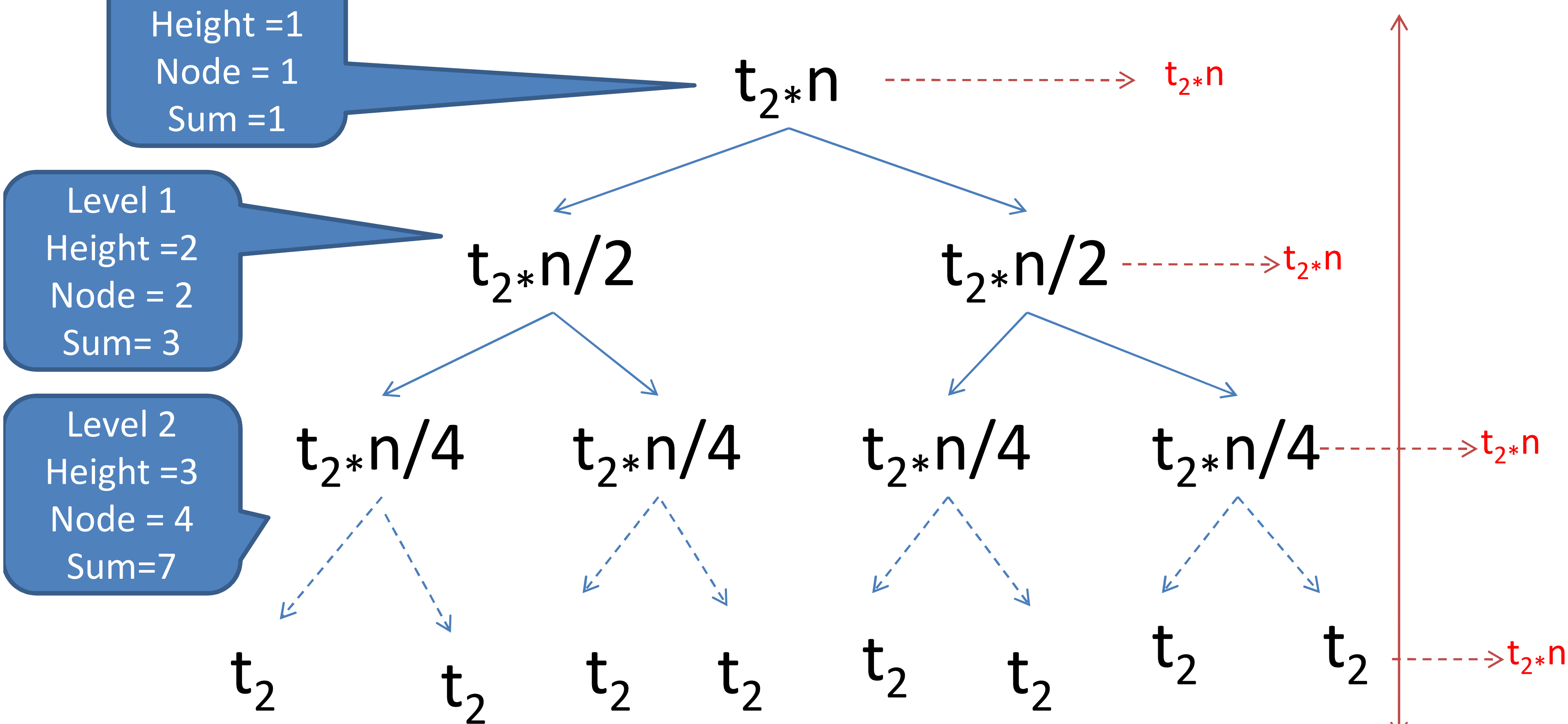
Conquer
$T2 = 2T(n/2)$

Combine
$T3 = t_3 * n$

$T(n) = t_1$ If n =1

$T(n) = 2T(n/2) + t_3 \times n$ If n >1

Analyze the Running time of merge-sort



Number of nodes in each level = 2^L
Number of cumulative nodes = 2^H -1
H = log_2 (N +1)

Analyze the Running time of merge-sort